Table 4.1

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 4.2

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(x)\,dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

Repeated application of this property yields

$$\frac{d^n f}{dt^n} \Longleftrightarrow (j\omega)^n F(\omega) \tag{4.48}$$

The time-integration property [Eq. (4.47)] has already been proved in Example 4.13.

■ **Example 4.14**

Using the time-differentiation property, find the Fourier transform of the triangle pulse $\Delta(\frac{t}{\tau})$ illustrated in Fig. 4.25a.

To find the Fourier transform of this pulse we differentiate the pulse successively, as illustrated in Fig. 4.25b and c. Because df/dt is constant everywhere, its derivative, d^2f/dt^2, is zero everywhere. But df/dt has jump discontinuities with a positive jump of $2/\tau$ at $t = \pm\frac{\tau}{2}$, and a negative jump of $4/\tau$ at $t = 0$. Recall that the derivative of a signal at a jump discontinuity is an impulse at that point of strength equal to the amount of jump. Hence, d^2f/dt^2, the derivative of df/dt, consists of a sequence of impulses, as depicted in Fig. 4.25c; that is,

$$\frac{d^2 f}{dt^2} = \frac{2}{\tau}[\delta(t+\tfrac{\tau}{2}) - 2\delta(t) + \delta(t-\tfrac{\tau}{2})] \tag{4.49}$$

Table 6.1

A Short Table of (Unilateral) Laplace Transforms

	$f(t)$	$F(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$
6	$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$
7	$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$
8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$
9a	$e^{-at}\cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
9b	$e^{-at}\sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$
10a	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{(r\cos\theta)s+(ar\cos\theta - br\sin\theta)}{s^2+2as+(a^2+b^2)}$
10b	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$
10c	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{As+B}{s^2+2as+c}$
	$r=\sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{c-a^2}}$	
	$b=\sqrt{c-a^2}$	
10d	$e^{-at}\left[A\cos bt+\frac{B-Aa}{b}\sin bt\right]u(t)$	$\frac{As+B}{s^2+2as+c}$
	$b=\sqrt{c-a^2}$	

Table 6.2

The Laplace Transform Properties

Operation	$f(t)$	$F(s)$
Addition	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
Scalar multiplication	$kf(t)$	$kF(s)$
Time differentiation	$\dfrac{df}{dt}$	$sF(s) - f(0^-)$
	$\dfrac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - \dot{f}(0^-)$
	$\dfrac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - s\dot{f}(0^-) - \ddot{f}(0^-)$
Time integration	$\displaystyle\int_{0^-}^{t} f(\tau)\,d\tau$	$\dfrac{1}{s}F(s)$
	$\displaystyle\int_{-\infty}^{t} f(\tau)\,d\tau$	$\dfrac{1}{s}F(s) + \dfrac{1}{s}\displaystyle\int_{-\infty}^{0^-} f(t)\,dt$
Time shift	$f(t-t_0)u(t-t_0)$	$F(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shift	$f(t)e^{s_0t}$	$F(s-s_0)$
Frequency differentiation	$-tf(t)$	$\dfrac{dF(s)}{ds}$
Frequency integration	$\dfrac{f(t)}{t}$	$\displaystyle\int_{s}^{\infty} F(z)\,dz$
Scaling	$f(at),\ a \geq 0$	$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Frequency convolution	$f_1(t)f_2(t)$	$\dfrac{1}{2\pi j}F_1(s) * F_2(s)$
Initial value	$f(0^+)$	$\lim_{s\to\infty} sF(s) \quad (n > m)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$ (poles of $sF(s)$ in LHP)

B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2+b^2}, \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1+\theta_2)}$$

B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \tfrac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \tfrac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos\left(x \pm \tfrac{\pi}{2}\right) = \mp\sin x$$

$$\sin\left(x \pm \tfrac{\pi}{2}\right) = \pm\cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

Table 10.4 Butterworth Polynomials.

Order *n*	Denominator polynomial $Q_N(s)$ in polynomial form
1	$1 + s$
2	$1 + \sqrt{2}s + s^2$
3	$1 + 2s + 2s^2 + s^3$
4	$1 + 2.613s + 3.414s^2 + 2.613s^3 + s^4$
5	$1 + 3.236s + 5.236s^2 + 5.236s^3 + 3.236s^4 + s^5$
6	$1 + 3.864s + 7.464s^2 + 9.141s^3 + 7.464s^4 + 3.864s^5 + s^6$
7	$1 + 4.494s + 10.103s^2 + 14.606s^3 + 14.606s^4 + 10.103s^5 + 4.494s^6 + s^7$
8	$1 + 5.126s + 13.138s^2 + 21.848s^3 + 25.691s^4 + 21.848s^5 + 13.138s^6 + 5.126s^7 + s^8$
Order *n*	**Denominator polynomial $Q_N(s)$ in factored form**
1	$1 + s$
2	$1 + \sqrt{2}s + s^2$
3	$(1 + s)(1 + s + s^2)$
4	$(1 + 0.76536s + s^2)(1 + 1.84776s + s^2)$
5	$(1 + s)(1 + 0.6180s + s^2)(1 + 1.6180s + s^2)$
6	$(1 + 0.5176s + s^2)(1 + \sqrt{2}s + s^2)(1 + 1.9318s + s^2)$
7	$(1 + s)(1 + 0.4450s + s^2)(1 + 1.2456s + s^2)(1 + 1.8022s + s^2)$
8	$(1 + 0.3986s + s^2)(1 + 1.1110s + s^2)(1 + 0.6630s + s^2)(1 + 1.9622s + s^2)$

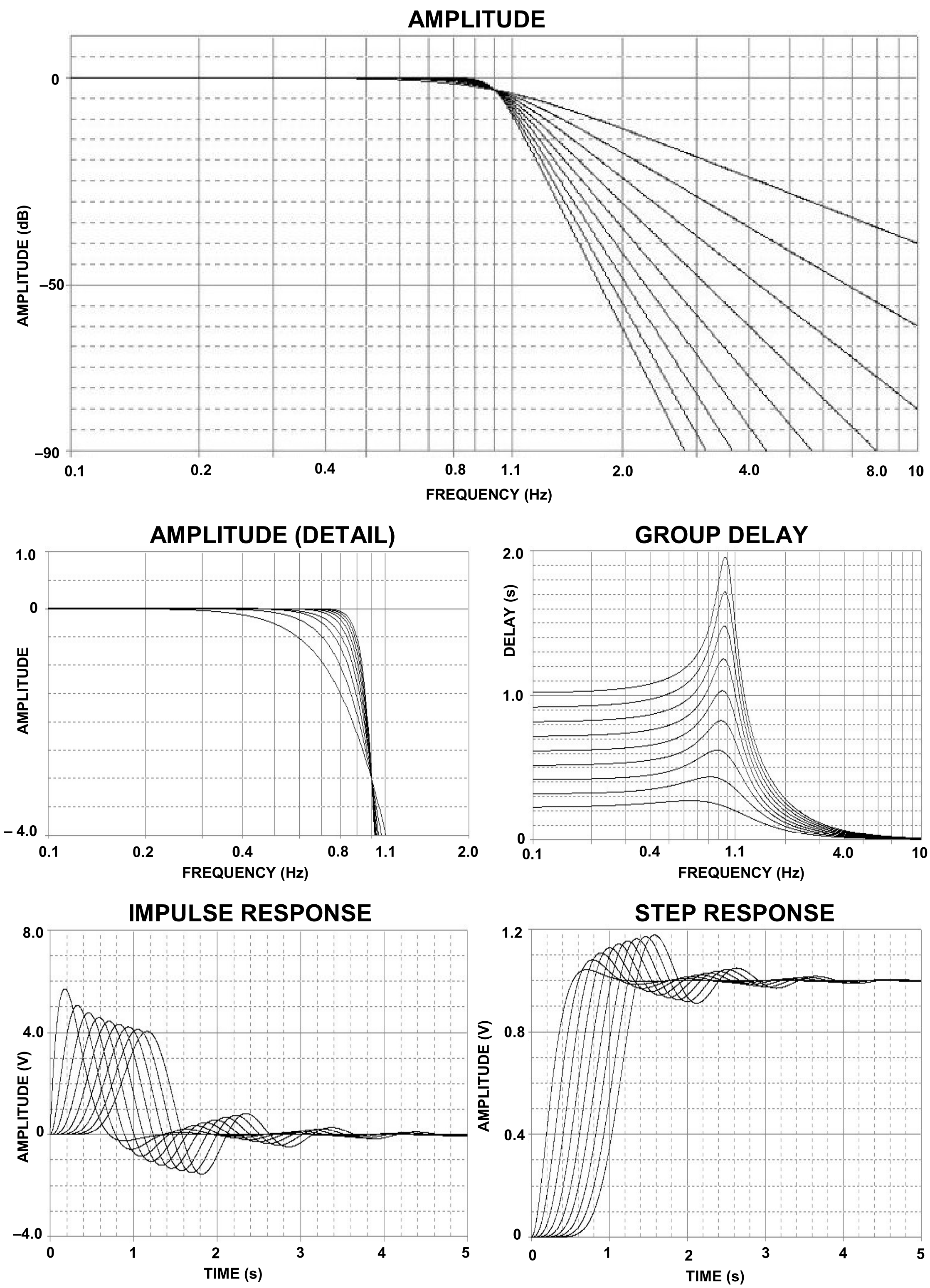

Figure 5-15: *Butterworth response*

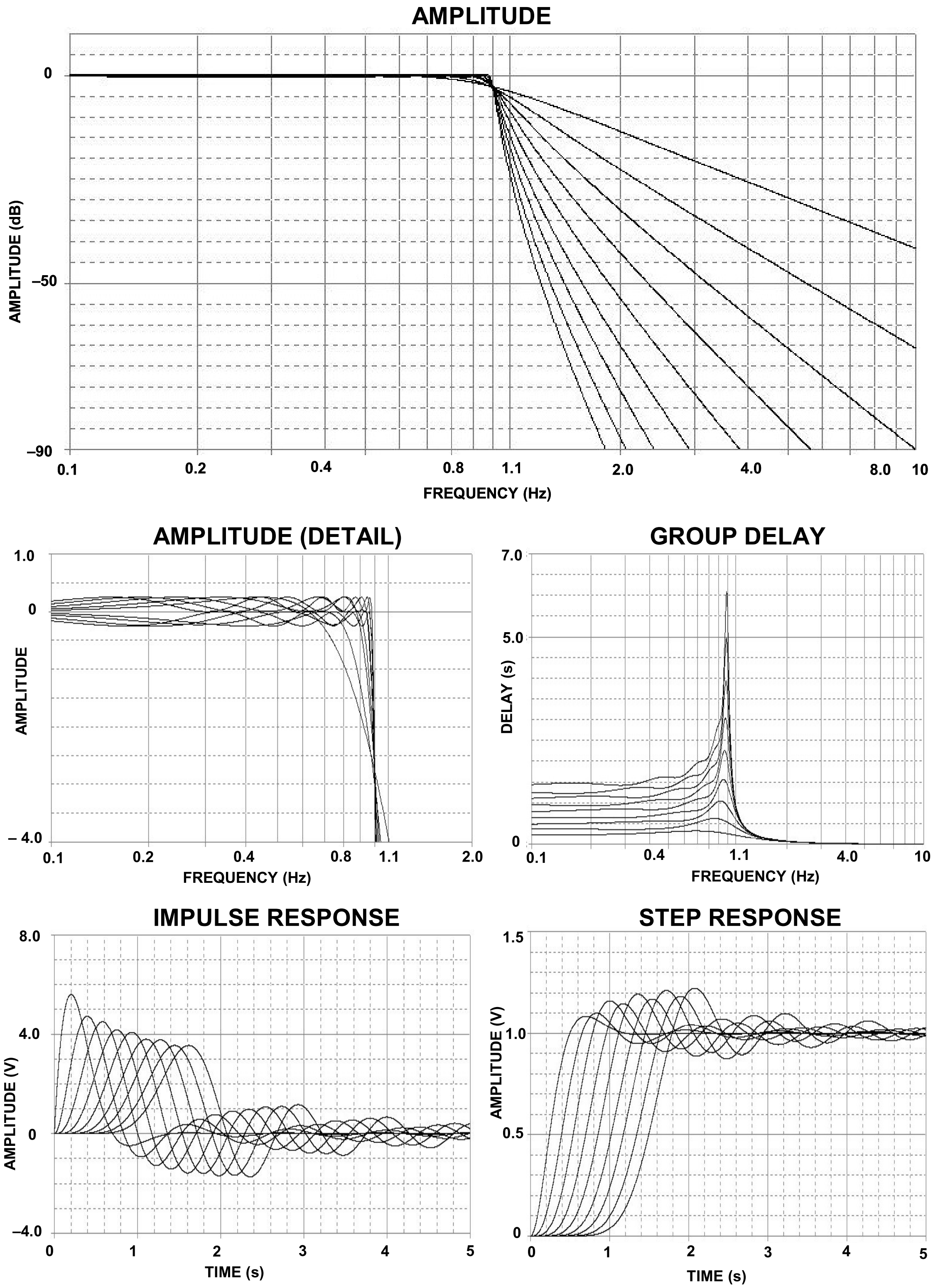

Figure 5-18: *0.25dB Chebyshev Response*

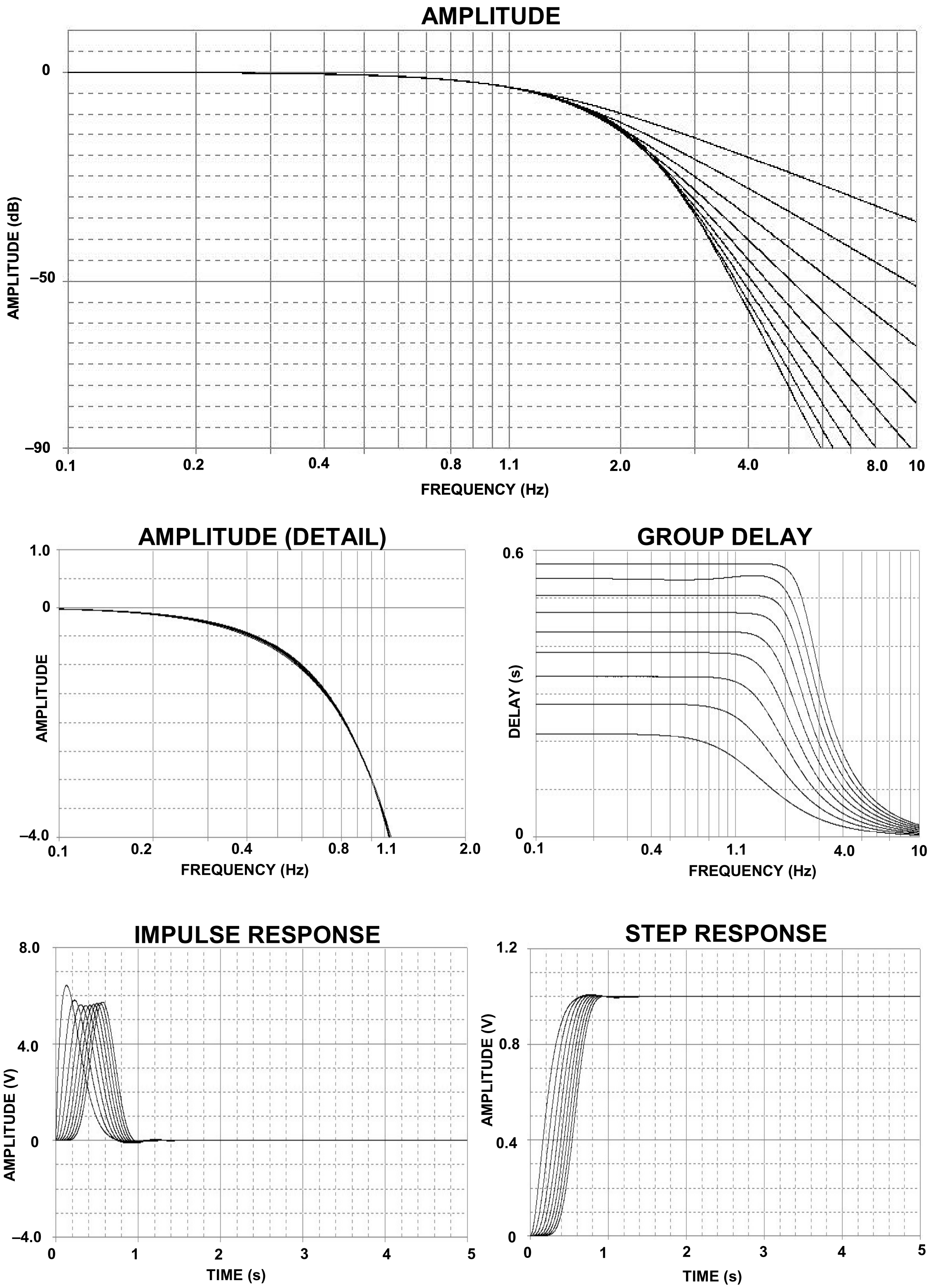

Figure 5-21: *Bessel Response*