Math Review

for Analog Signal Processing

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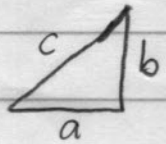
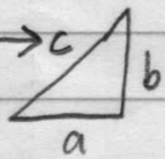
1.) Right Triangles

Pythagoras' theorem:

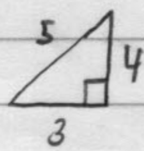
If 

is a right triangle, then $a^2+b^2=c^2$

Hypotenuse



is a right triangle $\Longrightarrow a^2+b^2=c^2$
$\Longleftarrow$

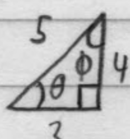


$$\Rightarrow 3^2+4^2=5^2$$
$$9+16=25$$
$$25=25 \checkmark$$

Trig: $\sin\theta = \dfrac{opp}{hyp}$ $\quad \cos\theta = \dfrac{adj}{hyp}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}} = \frac{opp}{adj}$$

inverses: $\theta = \sin^{-1}\dfrac{opp}{hyp}$ $\quad \theta = \cos^{-1}\dfrac{adj}{hyp}$ $\quad \theta = \tan^{-1}\dfrac{opp}{adj}$



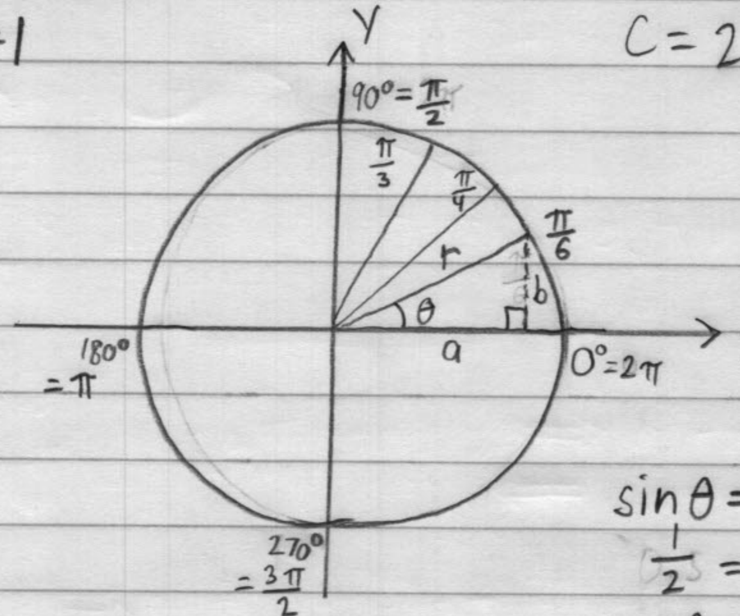
$$\theta = \sin^{-1}\frac{4}{5} = 53.13^\circ$$
$$\phi = \tan^{-1}\frac{3}{4} = 36.87^\circ$$

2.) Unit Circle

radius = 1

$C = 2\pi r$

$C = 2\pi \cdot 1 = 2\pi$



$\sin\theta = \frac{b}{r}$

$b = r\sin\theta$

$\cos\theta = \frac{a}{r}$

$a = r\cos\theta$

$r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\frac{b}{a}$

$\sin\theta = \frac{b}{1}$

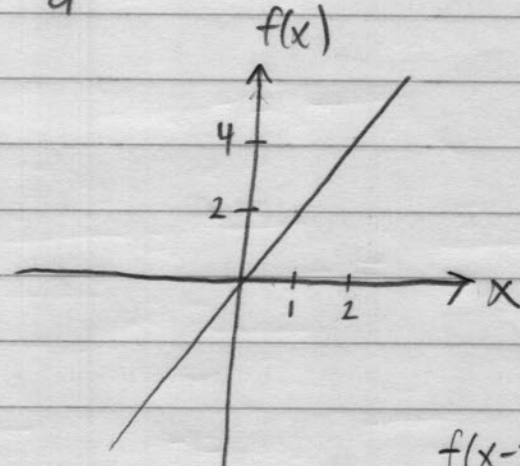
$\frac{1}{2} = b$

$\cos\theta = \frac{a}{1}$

$\frac{\sqrt{3}}{2} = a$

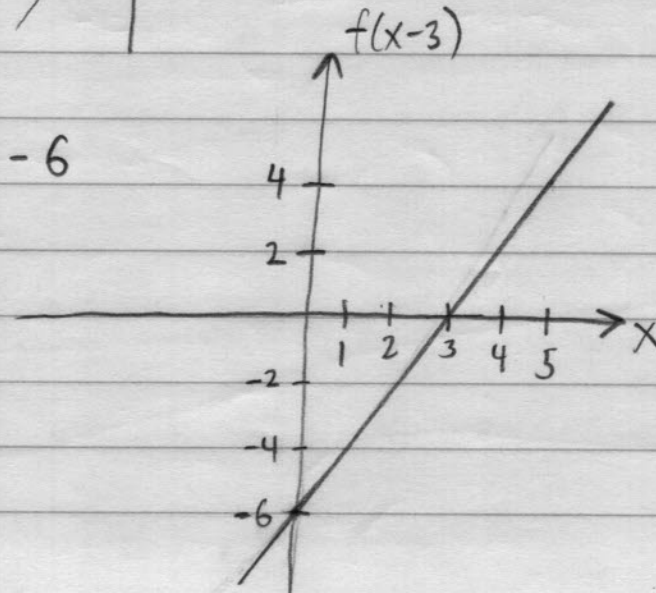
3.) Functions

$f(x) = 2x$



$f(x-3) = 2(x-3) = 2x-6$

Shifts function to the right by 3



Scaling: $f(x) = 3x-2$

$4f(x) = 4(3x-2)$

$= 12x-8$

Reflecting: $f(-x) = -3x-2$ Reflect on y (horizontal)

$-f(x) = -3x+2$ Reflect on x (vertical)

$f(x) = 2x$

$0 = 2x$

$x = 0$

$f(x-3) = 2x-6$

$0 = 2x-6$

$6 = 2x$

$x = 3$

$f(x) = x^2+2x-3$

$x^2+2x-3=0$

$(x+3)\cdot(x-1)=0$

$x+3=0$

$x=-3$

$x-1=0$

$x=1$

Two solutions, roots, zeros

Order of polynomial = # of solutions

$f(x) = x^2+2x+3$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$f(x) = ax^2+bx+c$

$x = \dfrac{-2 \pm \sqrt{4-12}}{2}$

$= \dfrac{-2 \pm \sqrt{-8}}{2} = \dfrac{-2 \pm 2\sqrt{-2}}{2}$

$= -1 \pm \sqrt{-2} = -1 \pm \sqrt{2}\cdot\sqrt{-1} \Longrightarrow$

$= -1 \pm \sqrt{2}j$

Two solutions

1.) $-1+\sqrt{2}j$

2.) $-1-\sqrt{2}j$

$f(x) = \dfrac{x^2+2x-3}{x^2-5x-6} = \dfrac{(x+3)(x-1)}{(x-6)(x+1)}$

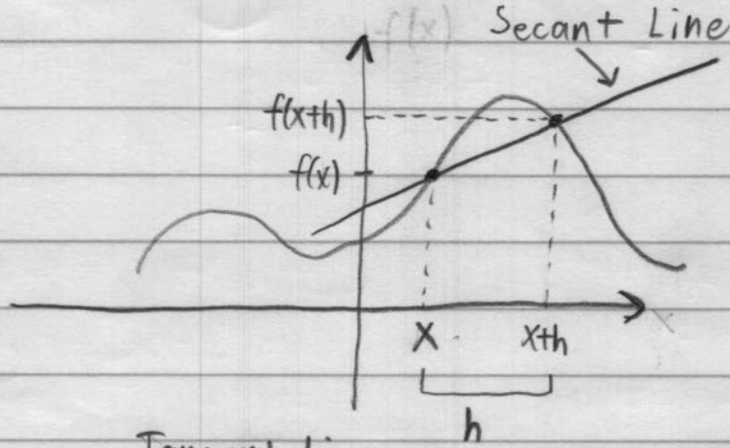
zeros

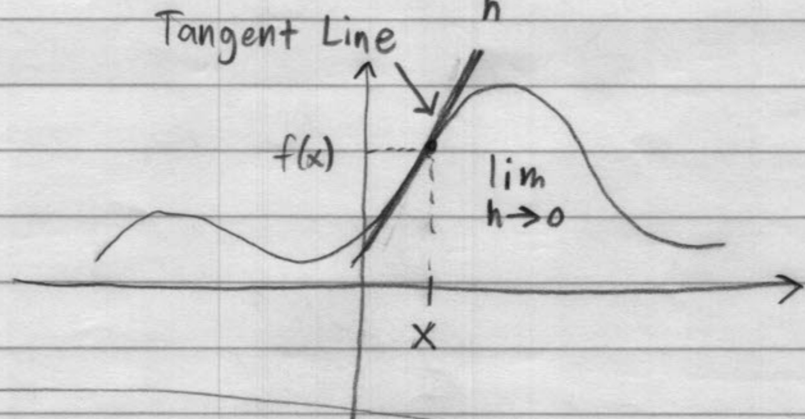
$x=-3, 1$

$x = 6, -1$

asymptotes

5.) Derivatives $\qquad \dfrac{d}{dx} f(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$



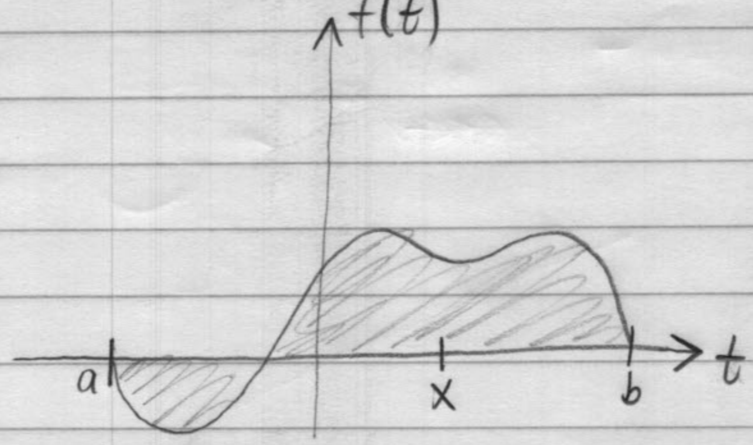


$$\boxed{\text{Notation: } \frac{d}{dx} f(x) = f'(x) = \frac{df}{dx}} \qquad f''(x) = \frac{d^2 f}{dx^2}$$

$$f(x) = 3x \qquad g(x) = 4x^2$$
$$f'(x) = 3 \qquad g'(x) = 8x$$

$$h(x) = f(x) + g(x) = 3x + 4x^2$$
$$h'(x) = f'(x) + g'(x) = 3 + 8x$$
$$h''(x) = f''(x) + g''(x) = 0 + 8 = 8$$

6.) Integrals



$$A = \int_a^b f(t)\,dt \qquad \text{Function of Area } F(x) = \int_a^x f(t)\,dt$$

$$F'(x) = f(x) \qquad \int_a^b f(x)\,dx = F(b) - F(a)$$

Fundamental Theorem of calculus

Examples:

$$\int_2^3 x^2\,dx = \frac{1}{3}x^3\Big|_2^3 = \frac{1}{3}(3^3 - 2^3) = \frac{1}{3}(27-8) = \frac{19}{3}$$

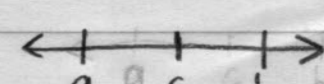
$$\int_{-1}^3 3\,dx = 3x\Big|_{-1}^3 = 3(3-(-1)) = 3\cdot 4 = 12$$

$$\int_0^4 x\,dx = \frac{1}{2}x^2\Big|_0^4 = \frac{1}{2}(16-0) = 8$$

$$\int_0^2 x\,dx + \int_2^4 x\,dx = \frac{1}{2}x^2\Big|_0^2 + \frac{1}{2}x^2\Big|_2^4 = \frac{1}{2}(2^2-0) + \frac{1}{2}(4^2-2^2)$$
$$= 2 + 6 = 8$$

$$\int_2^3 (x^2+3)\,dx = \int_2^3 x^2\,dx + \int_2^3 3\,dx = \frac{19}{3} + 3 = \frac{28}{3}$$

$$\int_0^\infty e^{-x}\,dx = -e^{-x}\Big|_0^\infty = -(0-1) = 1$$



$$\int_a^b (x+x^2)\,dx = \int_a^b x\,dx + \int_a^b x^2\,dx \qquad \int_a^b \sin x\,dx = \int_a^c \sin x\,dx + \int_c^b \sin x\,dx$$