Stephan Kulov

Analog Signal Processing

1.) Intro

Normally Requires ECE 225 (Circuit Analysis) & MATH 220 (Diff. Eqns.)

- 2 hour teardown of an ECE core course
- Very fundamental topics which are used in all fields of electronics. Usually is a prerequisite for most other ECE classes
- Math heavy, but will learn something regardless
- Goal is to avoid time-domain and use the frequency domain instead. Learn tools along the way

2.) Complex numbers (Real line + Img line = Complex plane)

abs value

$z = a + jb$ $\quad j = \sqrt{-1}$ $\quad j^2 = -1$ $\quad Re(z) = a$ $\quad Im(z) = b$

$z^* = a - jb$

Rectangular Form

Polar form: $z = |z| e^{j\angle z}$

$|z| = \sqrt{z \cdot z^*} = \sqrt{a^2 + b^2}$ $\quad \angle z = \tan^{-1} \frac{Im(z)}{Re(z)}$

$a = |z| \cos\angle z$ $\quad b = |z| \sin\angle z$

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

3.) Impedances

$V = IR$ $\quad V = IZ$ $\quad Z = R + \underbrace{X}_{\text{Reactance}}$ $\quad Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

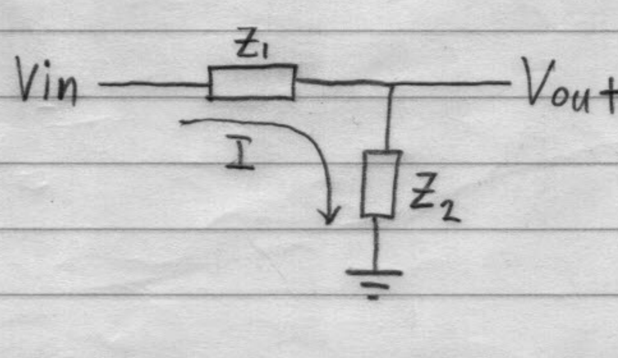
$X_c = \frac{1}{j\omega C} = \frac{1}{sC}$ $\quad X_L = j\omega L = sL$

Series

$Z_{RC} = R + \frac{1}{j\omega C} = R + \frac{1}{sC}$

$Z_{RL} = R + j\omega L = R + sL$

4.) Voltage Divider



$$V_{out} = V_{in} \frac{Z_2}{Z_1 + Z_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

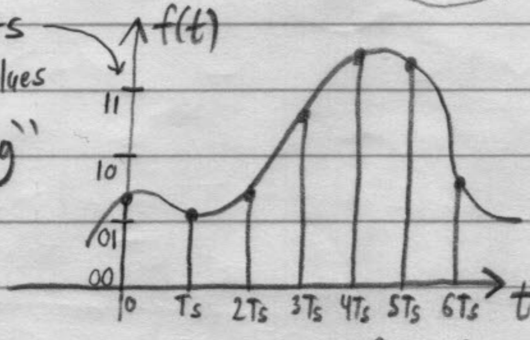
Essentially the same

These are Different

5.) Signals: Continuous-time, Discrete-time
Analog, Digital

2 bits
$2^2 = 4$ values

"Quantizing"
= Rounding to these values

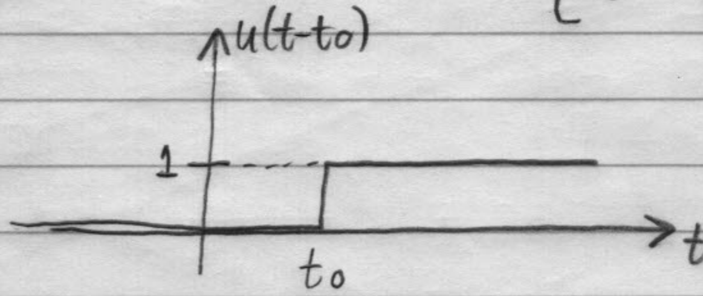


T_s = sampling time

Sampling a continuous-time, or analog, signal gives you a discrete-time signal.
Quantizing a discrete-time signal results in a digital signal.

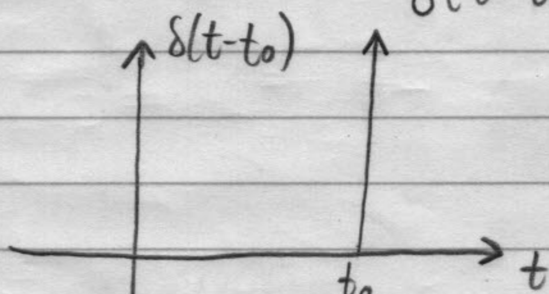
Aperiodic Signals:

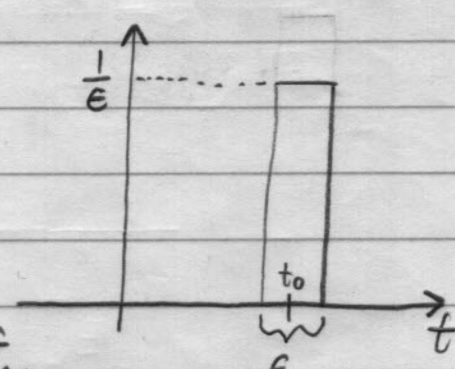
Unit Step $\quad u(t-t_0) = \begin{cases} 1, & t \geq t_0 \\ 0, & t < t_0 \end{cases}$



Causal signal always starts on or after $t=0$

Impulse function $\quad \delta(t-t_0) = \begin{cases} 0, & t \neq t_0 \\ \infty, & t = t_0 \end{cases}$





$$\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1$$

$$\epsilon \cdot \frac{1}{\epsilon} = 1$$

The act of smacking something really hard.

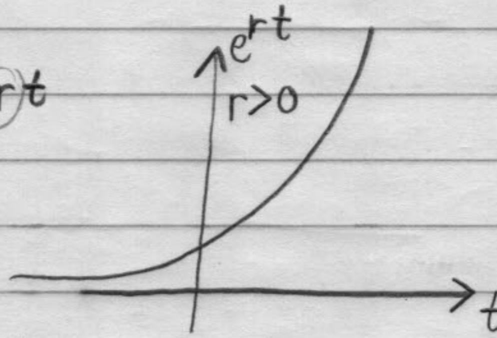
Sampling Property (or Sifting Property)

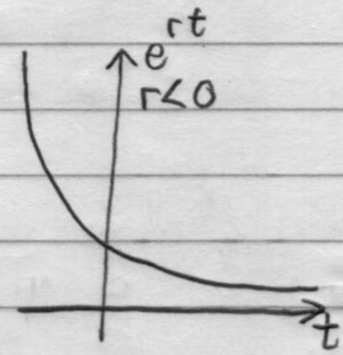
$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} f(t_0)\delta(t-t_0)\,dt = f(t_0)\overbrace{\int_{-\infty}^{\infty}\delta(t-t_0)\,dt}^{1} = f(t_0)$$

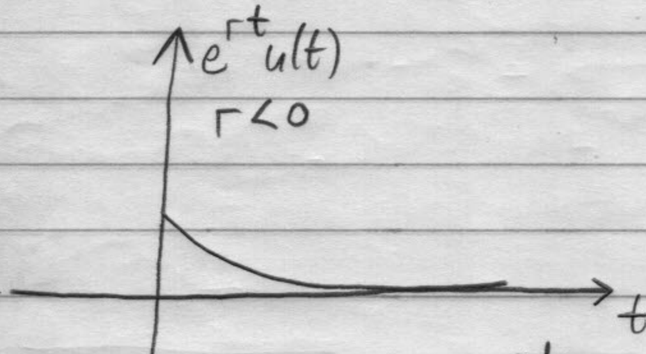
Exponentials e^{rt}

growth/decay rate





Common signal: $e^{rt}u(t)$, $r<0$



Periodic Signals $\cos(\underset{\text{rad/s}}{\omega_0} t)$

$f(t)=f(t-T_0)$

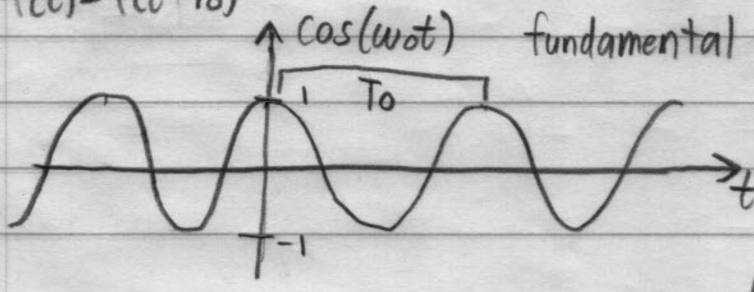
fundamental frequency

$T_0 = \dfrac{1}{f_0} \Rightarrow f_0 = \dfrac{1}{T_0}$

fundamental Period

$\omega_0 = 2\pi f_0 = \dfrac{2\pi}{T_0}$

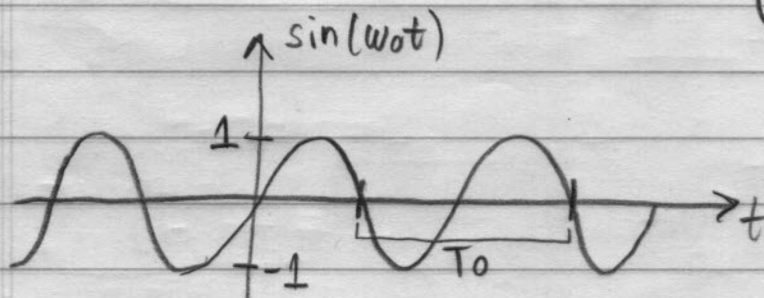
Even
$f(-t)=f(t)$



$170\cos(2\pi 60t)u(t)$

Flipping a switch at $t=0$ to turn on a wall outlet.

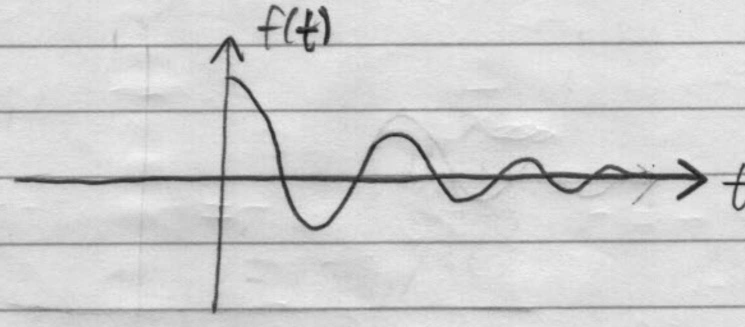
Odd
$f(-t)=-f(t)$



$$f(t) = e^{rt}\cos(\omega_0 t)\,u(t)$$

$$r < 0$$

Musical instruments such as string or percussion instruments



6.) Linear Systems

LTI CTS ← Linear Time-Invariant Continuous-time system

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

Linearity (Superposition)

LTI CTS

$$\alpha x_1(t) + \beta x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow \alpha y_1(t) + \beta y_2(t)$$

Time Invariance

LTI CTS

$$x(t-t_0) \longrightarrow \boxed{h(t)} \longrightarrow y(t-t_0)$$

Impulse response $h(t)$

Smacking a steel beam analogy

Act of smacking: $\delta(t)$

Measure of vibrations due to smacking: $h(t)$

LTI CTS

$$\delta(t) \longrightarrow \boxed{?} \longrightarrow h(t)$$

7.) Convolution – So annoying that there are youtube videos of people trying to do ~~them~~ problems in record time

LTI CTS

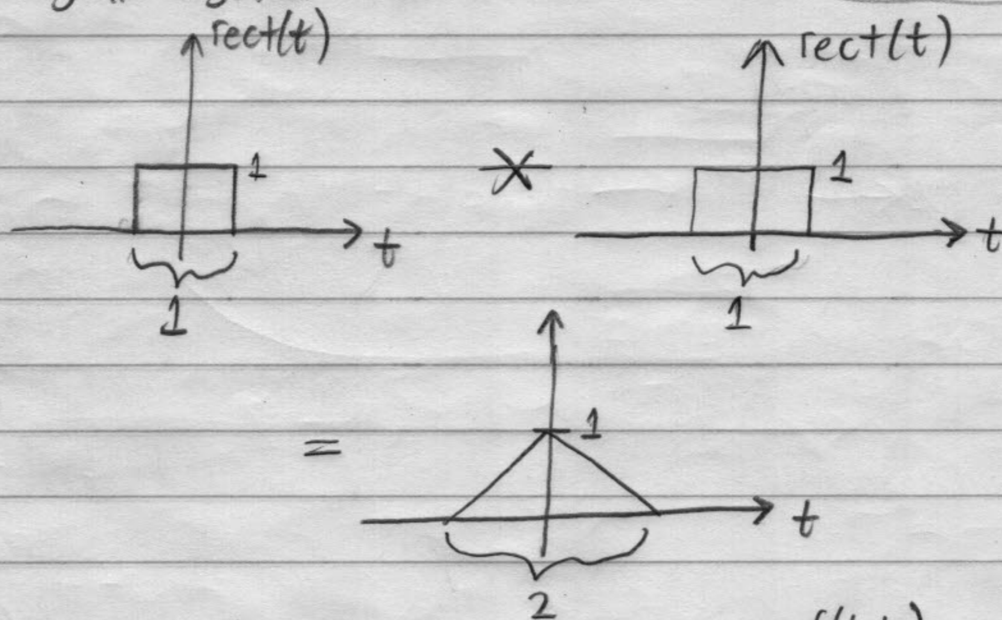
$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

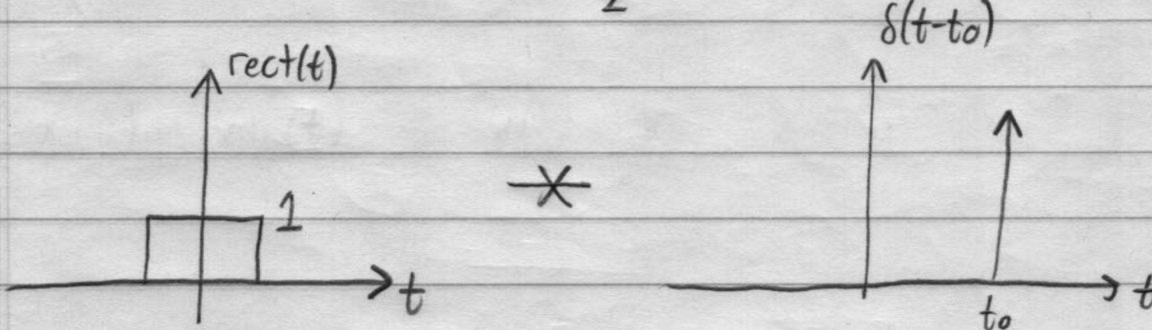
Convolution

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\cdot x_2(t-\tau)\,d\tau = x_2(t) * x_1(t)$$

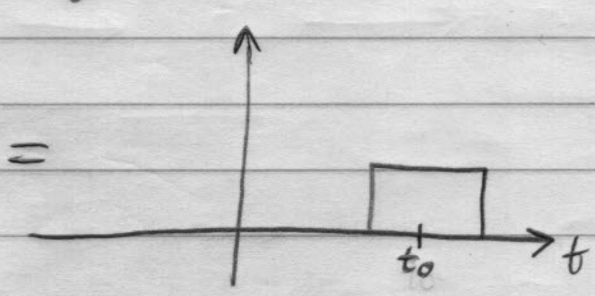
Like "smearing" two signals together

Flipping and sliding over one another.





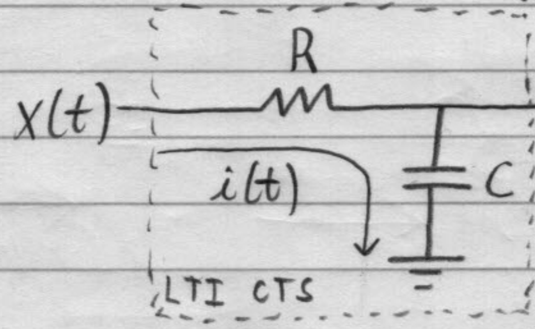
Convolution with an impulse recenters the signal on where the impulse was



$$v_c(t) = \frac{1}{c}\int_{-\infty}^{t} i_c(\tau)d\tau \qquad v_L(t) = L i_L'(t)$$

$$i_c(t) = C v_c'(t) \qquad i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)d\tau$$

8.) RC Circuit
(Lowpass Filter)



$i_R(t) = i_c(t)$

No initial conditions
$y(0^-) = 0$

$$\frac{x(t)-y(t)}{R} = Cy'(t) \implies y'(t) = \frac{x(t)-y(t)}{RC}$$

Differential equation describing the system $\longrightarrow \boxed{y'(t) + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)}$ $\quad h(t) = \frac{1}{RC}y_n(t)\cdot u(t)$

Homogeneous Equation $\longrightarrow y_n'(t) + \frac{1}{RC}y_n(t) = 0$

Characteristic Equation $\longrightarrow r + \frac{1}{RC} = 0 \implies r = -\frac{1}{RC}$

$$y_n(t) = c\,e^{-\frac{t}{RC}}$$

Generic initial condition used when finding the impulse response $\longrightarrow$ $y_n(0) = 1 = Ce^{-\frac{0}{RC}} \implies c = 1$

$$y_n(t) = e^{-\frac{t}{RC}}$$

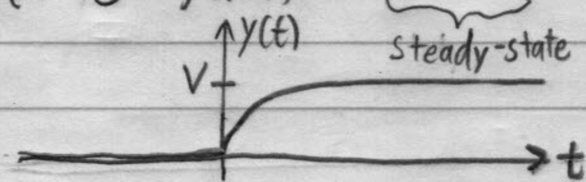
$$h(t) = \frac{1}{RC}e^{\frac{-t}{RC}}u(t)$$

Think of impulse response as the output of the circuit as if lightning hit the input of the circuit

$$x(t) = Vu(t) \qquad y(t) = Vu(t) * h(t) = \int_{-\infty}^{\infty} Vu(t-\tau)\cdot\frac{1}{RC}e^{-\frac{\tau}{RC}}d\tau$$

$$y(t) = \int_0^t \frac{V}{RC}e^{-\frac{\tau}{RC}}d\tau = -Ve^{-\frac{\tau}{RC}}\Big|_0^t = -V(e^{-\frac{t}{RC}}-1)u(t)$$

$$= V(1-e^{-\frac{t}{RC}})u(t) = \underbrace{Vu(t)}_{\text{steady-state}} - \underbrace{Ve^{-\frac{t}{RC}}u(t)}_{\text{transient}}$$



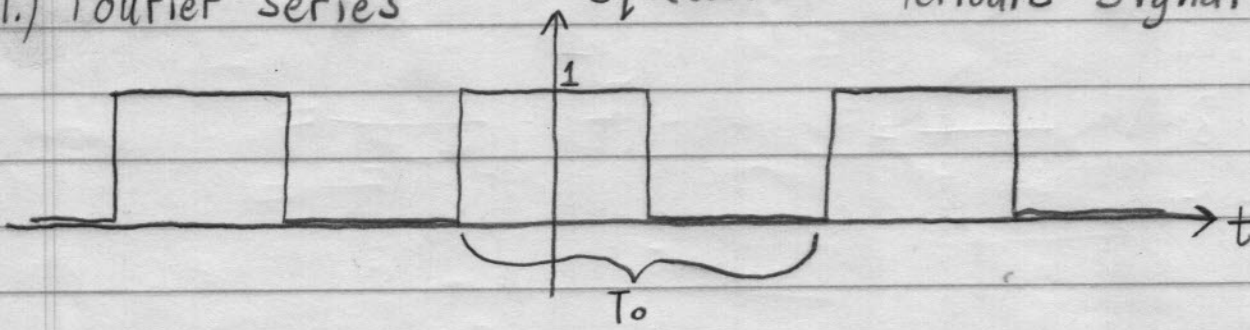
Synthesis Equation

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

Analysis Equation

$$X_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt$$

9.) Fourier Series $sqw(\omega_0 t)$ Periodic Signal



$$X_n = \frac{1}{T_0} \int_{-\frac{T_0}{4}}^{\frac{3T_0}{4}} sqw(\omega_0 t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} e^{-jn\omega_0 t} dt = -\frac{1}{T_0 jn\omega_0} e^{-jn\omega_0 t} \Big|_{-\frac{T_0}{4}}^{\frac{T_0}{4}}$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$= -\frac{1}{T_0 jn\omega_0} \left(e^{-jn\omega_0 \frac{T_0}{4}} - e^{jn\omega_0 \frac{T_0}{4}} \right)$$

$$= -\frac{1}{jn 2\pi} \left(e^{-jn\frac{\pi}{2}} - e^{jn\frac{\pi}{2}} \right) = \frac{1}{jn2\pi} \left(e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}} \right)$$

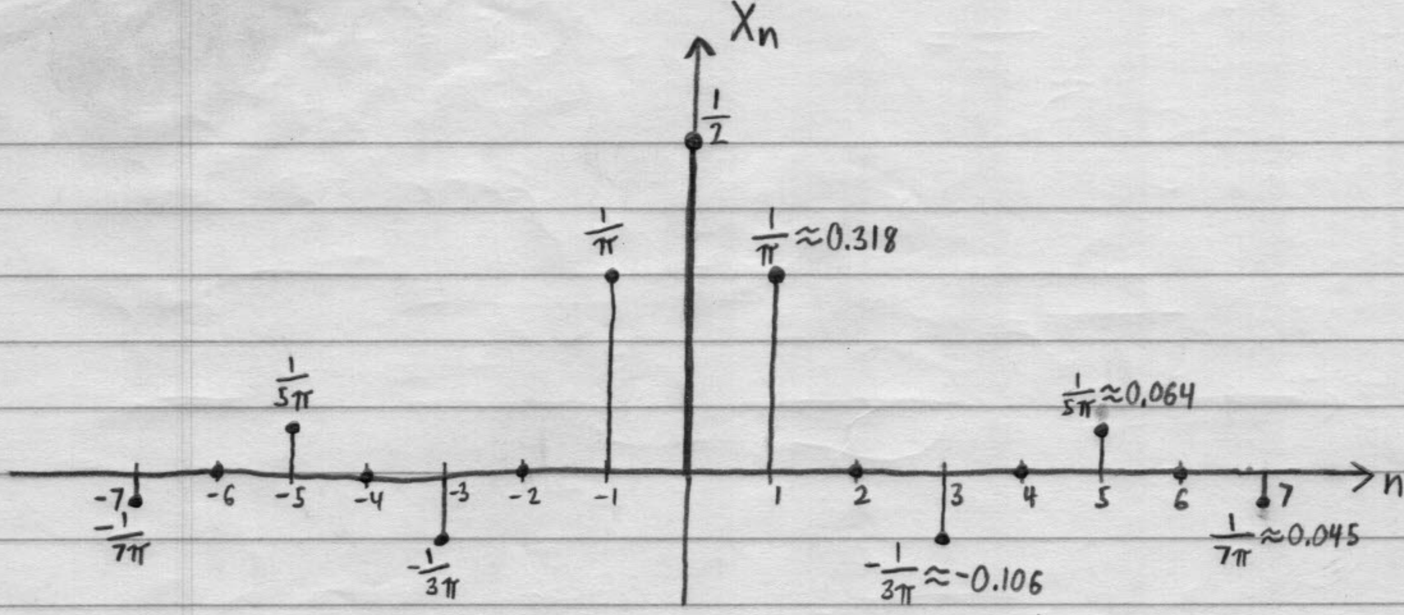
$$= \frac{1}{n\pi} \frac{\left(e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}} \right)}{2j} = \frac{\sin(n\frac{\pi}{2})}{n\pi}$$

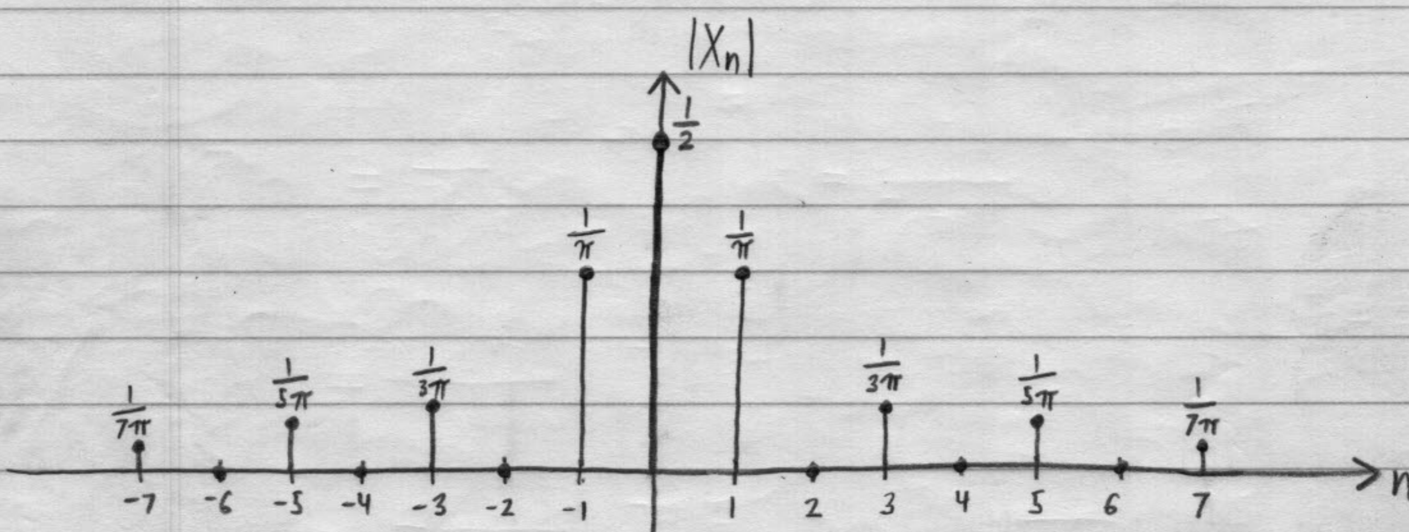
$$X_0 = \frac{1}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} 1\, dt = \frac{1}{T_0} t \Big|_{-\frac{T_0}{4}}^{\frac{T_0}{4}} = \frac{1}{T_0} \left(\frac{T_0}{4} + \frac{T_0}{4} \right) = \frac{1}{T_0} \cdot \frac{T_0}{2}$$

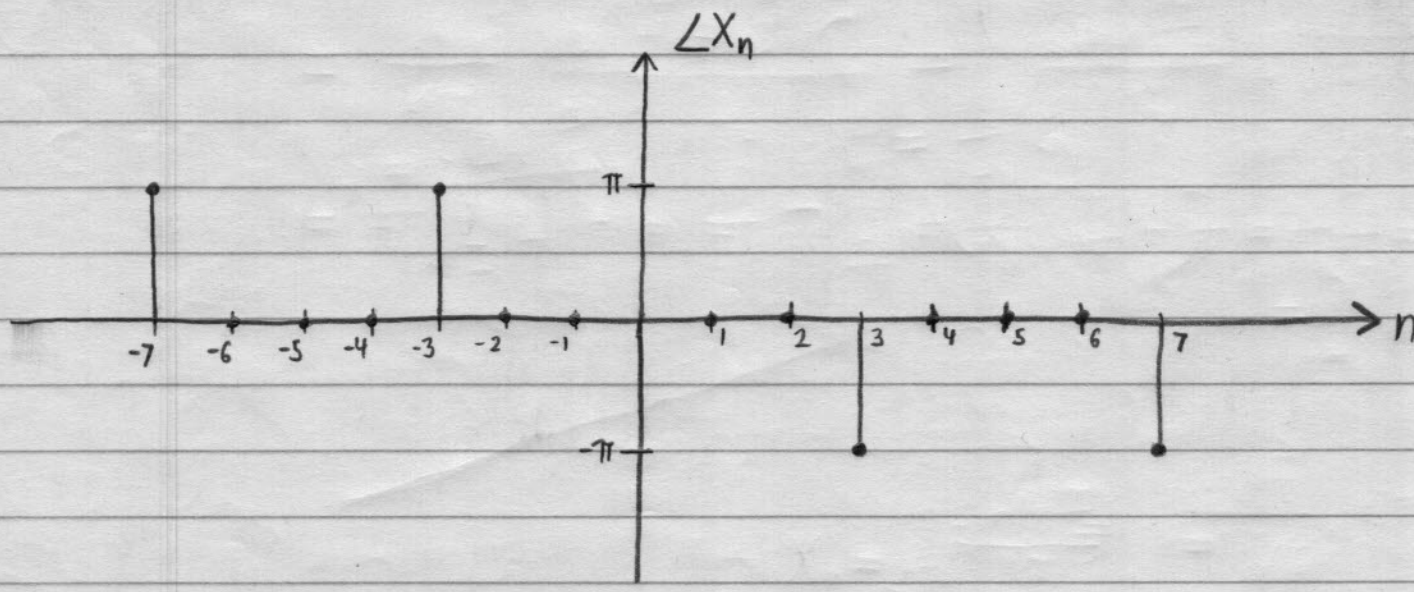
$$= \frac{1}{2}$$

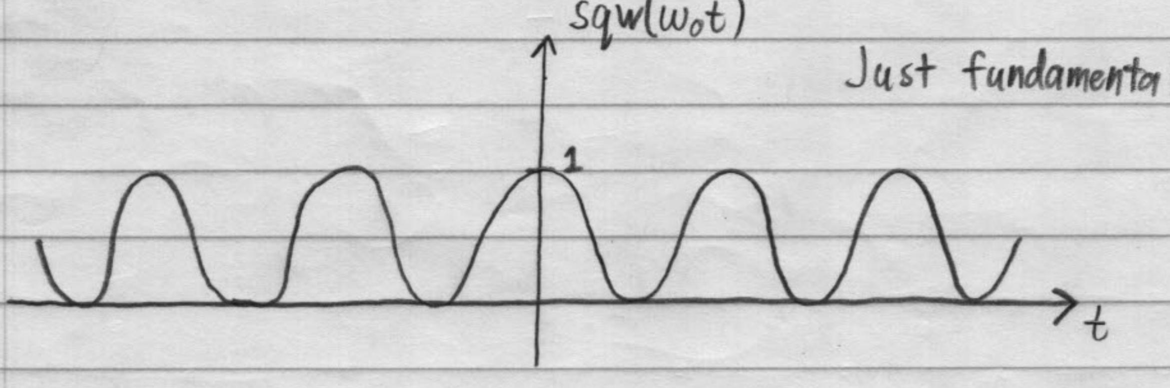
$$\boxed{X_n = \frac{\sin(n\frac{\pi}{2})}{n\pi} \qquad X_0 = \frac{1}{2}}$$

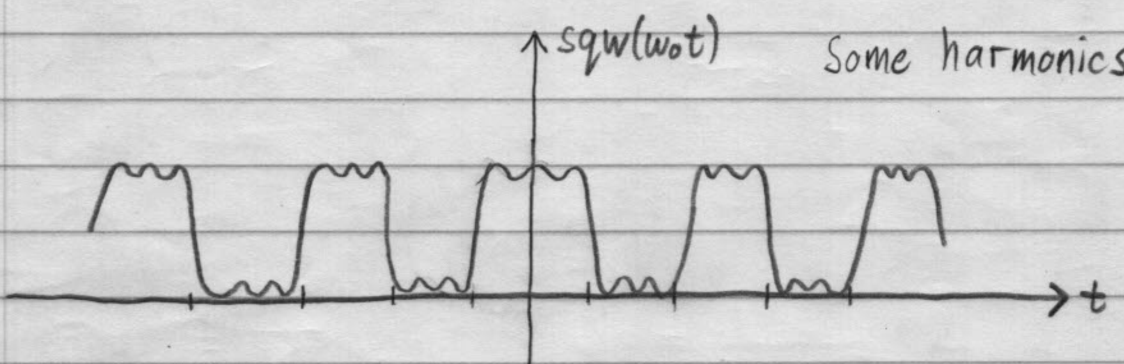
$$X_n = \begin{cases} \frac{\sin(n\frac{\pi}{2})}{n\pi}, & n \neq 0 \\ \frac{1}{2}, & n = 0 \end{cases}$$

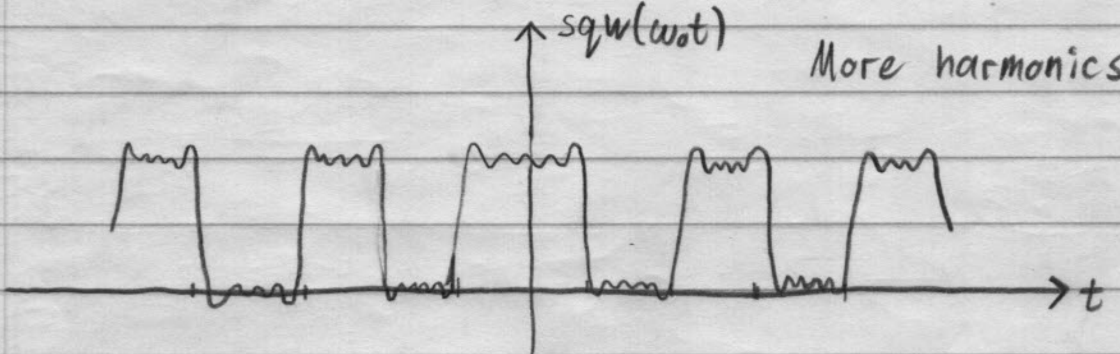

X_n
$\frac{1}{2}$
$\frac{1}{\pi}$
$\frac{1}{\pi} \approx 0.318$
$\frac{1}{5\pi}$
$\frac{1}{5\pi} \approx 0.064$
$-\frac{1}{7\pi}$
$-\frac{1}{3\pi}$
$-\frac{1}{3\pi} \approx -0.106$
$\frac{1}{7\pi} \approx 0.045$
-7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7
n

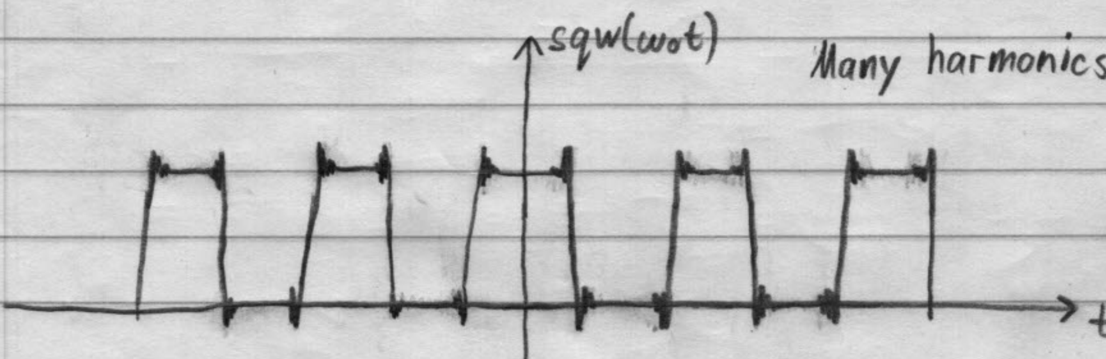

$|X_n|$
$\frac{1}{2}$
$\frac{1}{\pi}$
$\frac{1}{\pi}$
$\frac{1}{7\pi}$
$\frac{1}{5\pi}$
$\frac{1}{3\pi}$
$\frac{1}{3\pi}$
$\frac{1}{5\pi}$
$\frac{1}{7\pi}$
-7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7
n


$\angle X_n$
π
$-\pi$
-7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7
n









Gibbs Oscillation

Discuss Bandwidth
10MHz for Solderless Breadboard

$$sqw(\omega_0 t) = \frac{1}{5\pi}e^{-j5\omega_0 t} - \frac{1}{3\pi}e^{-j3\omega_0 t} + \frac{1}{\pi}e^{-j\omega_0 t} + \frac{1}{2}$$

$$+ \frac{1}{\pi}e^{j\omega_0 t} - \frac{1}{3\pi}e^{j3\omega_0 t} + \frac{1}{5\pi}e^{j5\omega_0 t} + \cdots$$

$$sqw(\omega_0 t) = \frac{1}{2} + \frac{1}{\pi}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) - \frac{1}{3\pi}\left(e^{j3\omega_0 t} + e^{-j3\omega_0 t}\right) + \frac{1}{5\pi}\left(e^{j5\omega_0 t} + e^{-j5\omega_0 t}\right) + \cdots$$

$$sqw(\omega_0 t) = \frac{1}{2} + \frac{2}{\pi}\cos(\omega_0 t) - \frac{2}{3\pi}\cos(\omega_0 t) + \frac{2}{5\pi}\cos(\omega_0 t) + \cdots$$

10.) Fourier Transform Aperiodic signals

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} \qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

$$\mathcal{F}\{x(t)\} = X(j\omega) \qquad \mathcal{F}^{-1}\{X(j\omega)\} = x(t)$$

$$x(t) \xleftrightarrow{\mathcal{F}} X(j\omega)$$

Linearity: $\alpha x_1(t) + \beta x_2(t) \xleftrightarrow{\mathcal{F}} \alpha X_1(j\omega) + \beta X_2(j\omega)$

Time shift: $x(t-t_0) \xleftrightarrow{\mathcal{F}} X(j\omega)e^{-j\omega t_0}$

Freq shift: $x(t)e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} X(j(\omega-\omega_0))$

Time differentiation: $\frac{d^n x}{dt^n} \xleftrightarrow{\mathcal{F}} (j\omega)^n X(j\omega)$

Convolution: $x_1(t) * x_2(t) \xleftrightarrow{\mathcal{F}} X_1(j\omega) \cdot X_2(j\omega)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 dw$$

Parseval's Theorem

$$h(t) = \frac{1}{RC} e^{\frac{-t}{RC}} u(t)$$

Impulse Response of Previous Circuit

LTI CTS

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

Discuss support for time v. freq.

$$Y(jw) = X(jw) \cdot H(jw)$$

$$\frac{Y(jw)}{X(jw)} = H(jw) \longleftarrow \text{Frequency Response}$$

$$H(jw) = \frac{1}{RC} \frac{1}{\frac{1}{RC} + jw} = \frac{1}{1 + jwRC}$$

$$\frac{1}{1 + jwRC} \frac{(1 - jwRC)}{(1 - jwRC)} = \frac{1 - jwRC}{1 + (RCw)^2} = \frac{1}{1 + (RCw)^2} - j \frac{RCw}{1 + (RCw)^2}$$

$$|H(jw)| = \sqrt{\left(\frac{1}{1 + (RCw)^2}\right)^2 + \left(\frac{RCw}{1 + (RCw)^2}\right)^2}$$

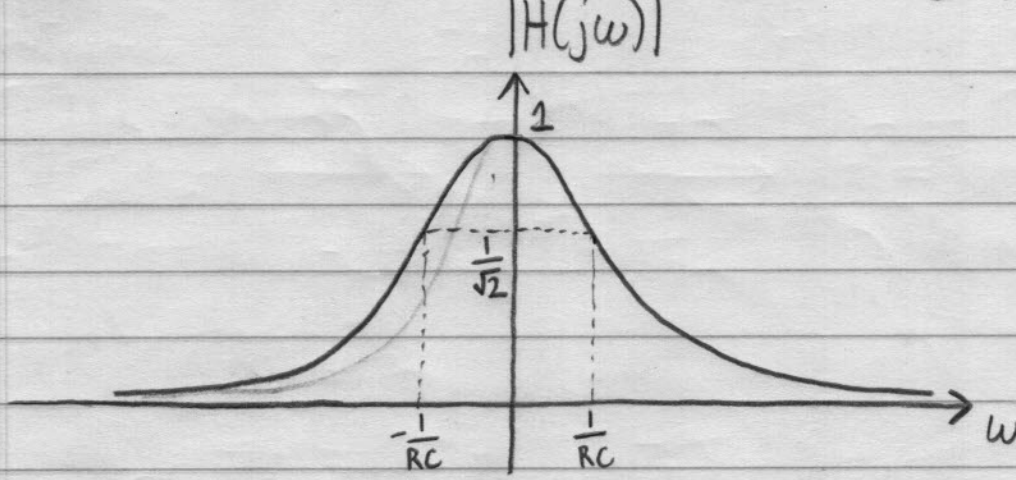
$$= \sqrt{\frac{1 + RCw^2}{(1 + (RCw)^2)^2}} = \frac{1}{\sqrt{1 + (RCw)^2}}$$

$$\angle H(jw) = \tan^{-1} \frac{-\frac{RCw}{1 + (RCw)^2}}{\frac{1}{1 + (RCw)^2}} = \tan^{-1} - RCw = -\tan^{-1} RCw$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (RCw)^2}} \Rightarrow 2 = 1 + (RCw)^2 \Rightarrow 1 = (RCw)^2$$

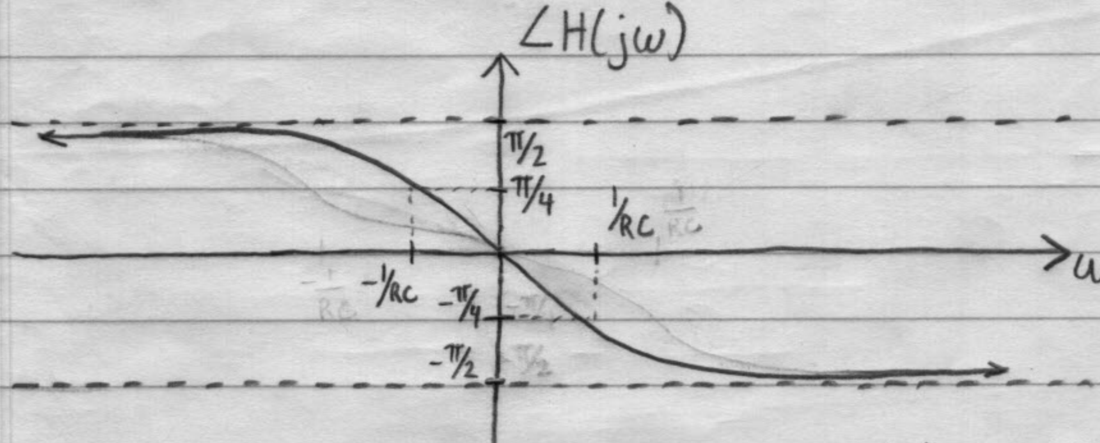
$$1 = RCw \Rightarrow w = \frac{1}{RC}$$

$|H(j0)| = 1$ $\lim_{\omega \to \infty} |H(j\omega)| = 0$



$\angle H(j\frac{1}{RC}) = -\tan^{-1} 1 = -\frac{\pi}{4}$ $\angle H(j0) = -\tan^{-1} 0 = 0$

$\lim_{\omega \to \infty} \angle H(j\omega) = \frac{-\pi}{2}$



Input: $\cos(\frac{1}{RC}t)$ $\Longrightarrow$ Output: $0.707\cos(\frac{1}{RC}t - \frac{\pi}{4})$

Lowpass Filter with $\omega_c = \frac{1}{RC}$ & $\angle H(j\omega_c) = -\frac{\pi}{4}$

$$20\log\frac{1}{\sqrt{2}} = 20\log\left(\frac{1}{2}\right)^{\frac{1}{2}} = 10\log\frac{1}{2} = -3.01 \text{ dB}$$

$$y'(t) + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

$$j\omega Y(j\omega) + \frac{1}{RC}Y(j\omega) = \frac{1}{RC}X(j\omega)$$

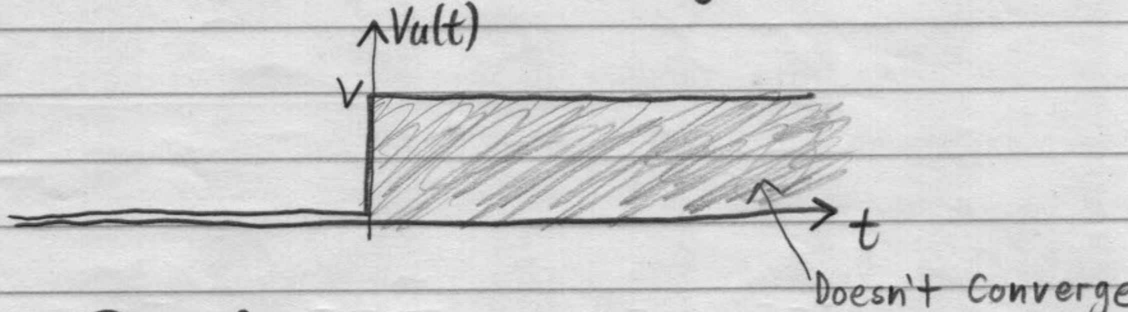
$$Y(j\omega)\left(j\omega + \frac{1}{RC}\right) = \frac{1}{RC}X(j\omega)$$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{1}{RC}\frac{1}{j\omega + \frac{1}{RC}}$$

$$H(j\omega) = \frac{1}{1 + RCj\omega}$$

$$Y(j\omega) = X(j\omega)\cdot H(j\omega)$$

$$= \frac{1}{1+RCj\omega}\cdot \mathcal{F}\{Vu(t)\} = \frac{V}{1+RCj\omega}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$



11.) Laplace Transform

$$\mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$\mathcal{F}\{x(t)e^{-\sigma t}\} = \int_{-\infty}^{\infty} x(t)e^{-\sigma t}e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} x(t)e^{-(\overbrace{\sigma+j\omega}^{s})t}\,dt$$

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

$$\mathcal{L}^{-1}\{X(s)\} = x(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{-st}\,ds$$

Convolution: $x_1(t) * x_2(t) \xleftrightarrow{\mathcal{L}} X_1(s)\cdot X_2(s)$

$$Y(s) = X(s)\cdot H(s) \Rightarrow \frac{Y(s)}{X(s)} = H(s)$$ ← Transfer Function

$$y'(t) + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

$$sY(s) + \frac{1}{RC}Y(s) = \frac{1}{RC}X(s)$$

$$Y(s)\left(s + \frac{1}{RC}\right) = \frac{1}{RC}X(s) \Rightarrow \frac{Y(s)}{X(s)} = H(s) = \frac{1}{1+RCs}$$

$$\mathcal{L}\{Vu(t)\} = \frac{V}{s}$$

$$Y(s) = \frac{V}{s} \cdot \frac{1}{1+RCs} = \frac{V}{s(1+RCs)}$$

$$\frac{V}{s(1+RCs)} = \frac{A}{s} + \frac{B}{1+RCs}$$

Partial Fraction Expansion

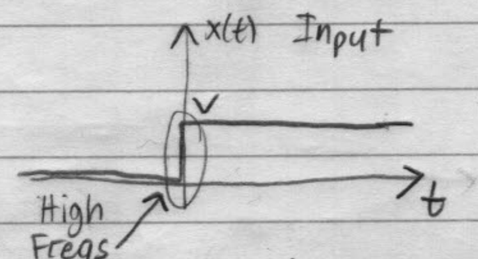
$$V = A(1+RCs) + Bs$$

$$s=0 \Rightarrow V = A(1+0)$$
$$V = A$$

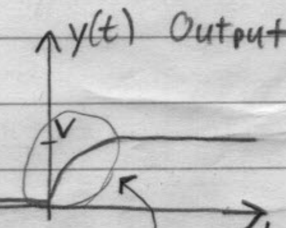
$$s = -\frac{1}{RC} \Rightarrow V = -\frac{B}{RC} \Rightarrow B = -VRC$$

$$\frac{V}{s(1+RCs)} = \frac{V}{s} + \frac{-VRC}{1+RCs} = \frac{V}{s} + \frac{1}{RC}\cdot\frac{-VRC}{\frac{1}{RC}+s}$$

$$Y(s) = \frac{V}{s} - \frac{V}{\frac{1}{RC}+s}$$



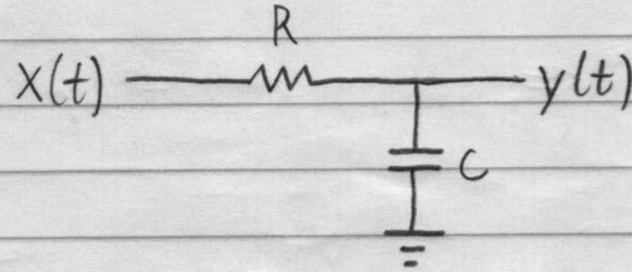
$$\underbrace{y(t) = Vu(t) - Ve^{-\frac{t}{RC}}u(t)}_{\text{Same Result as before}}$$

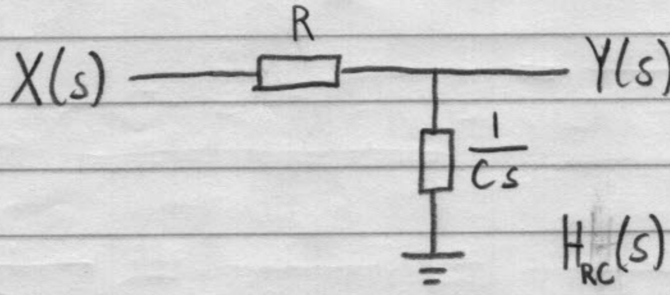


High Freqs are now removed (Filtered)

Transforming Networks

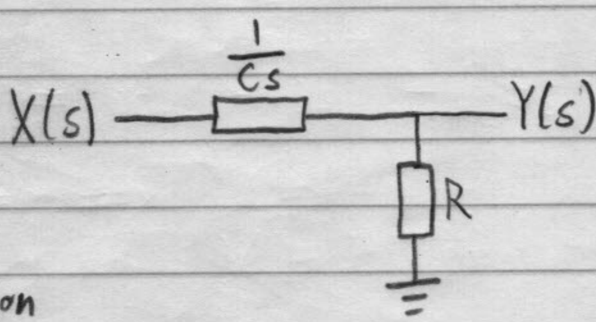
$Z_R = R \quad Z_C = \frac{1}{Cs} \quad Z_L = Ls$





$$Y(s) = X(s)\frac{\frac{1}{Cs}}{R+\frac{1}{Cs}}$$

$$H_{RC}(s) = \frac{Y(s)}{X(s)} = \frac{1}{RCs+1}$$



$$\frac{Y(s)}{X(s)} = \frac{R}{\frac{1}{Cs}+R} = \frac{RCs}{1+RCs}$$

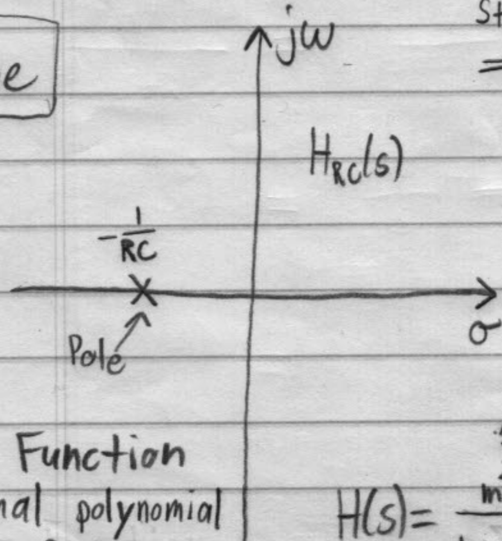
$$H_{CR}(s) = \frac{RCs}{1+RCs}$$

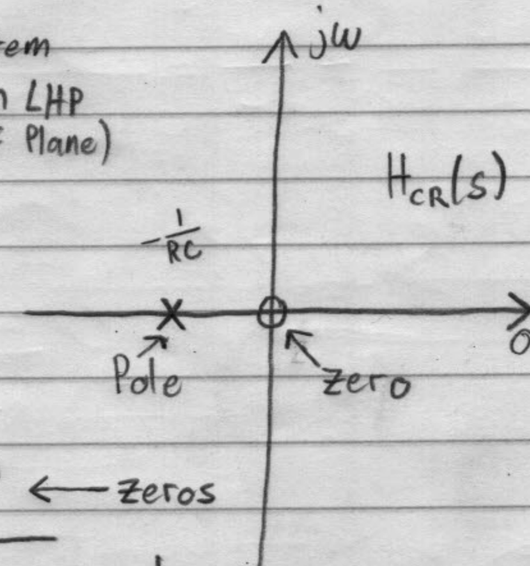
Transfer Function to Frequency Response

$$H_{RC}(s) = \frac{1}{RCs+1} \Longrightarrow H_{RC}(j\omega) = \frac{1}{RCj\omega+1}$$

$\boxed{\text{S-plane}}$

Stable system
$\Rightarrow$ Poles in LHP
(Left-Half Plane)

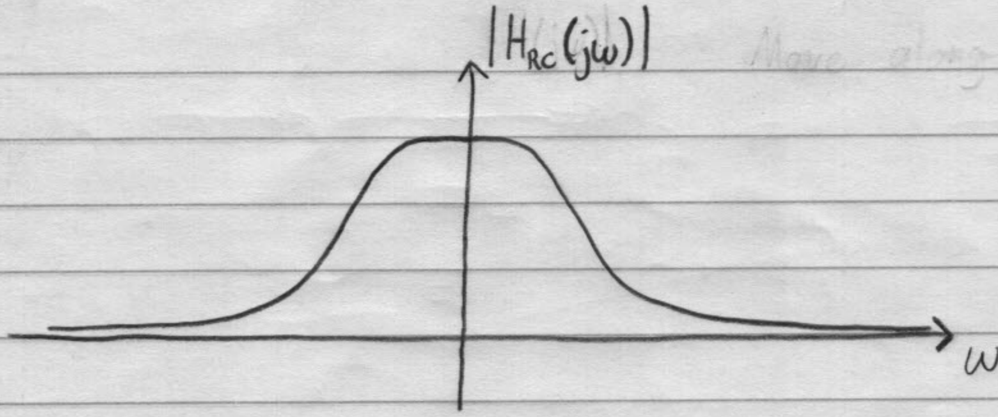


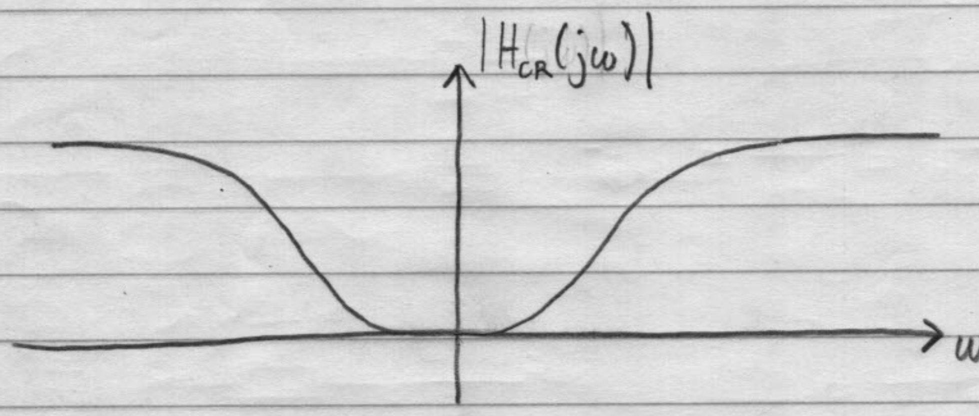


Transfer Function is rational polynomial in the form $\longrightarrow$

$$H(s) = \frac{\sum_{m=1}^{M} b_m s^m}{1+\sum_{n=1}^{N} a_n s^n} \begin{array}{l}\longleftarrow \text{zeros} \\ \longleftarrow \text{poles}\end{array}$$

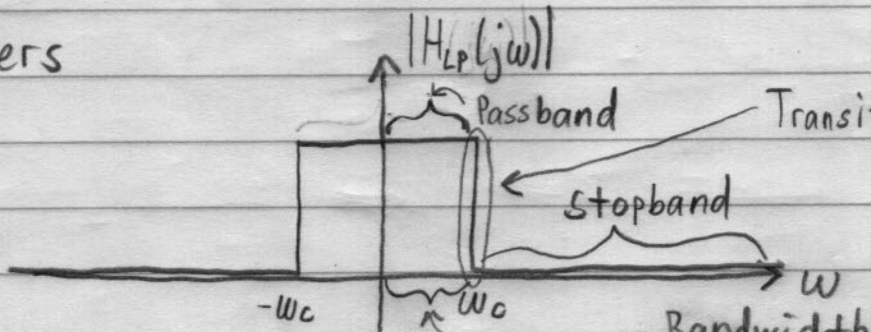
Rough sketch of Magnitude response by moving along $j\omega$-axis of s-plane:



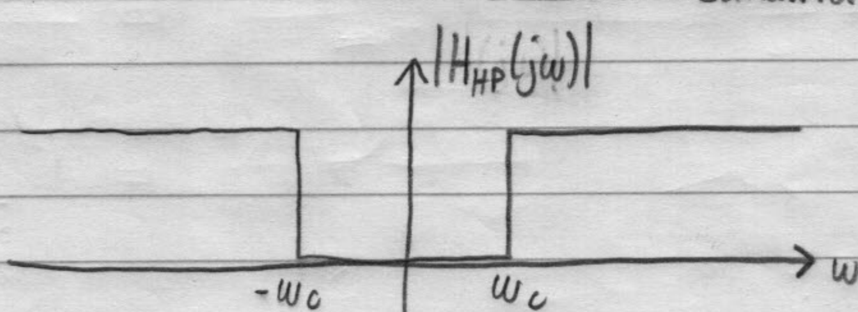


12.) Filters

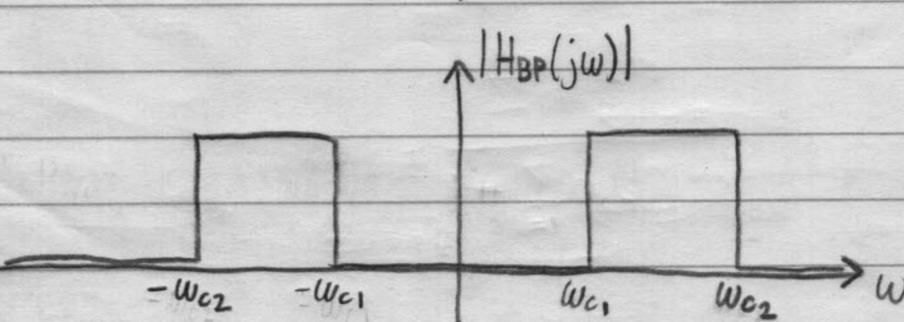
Ideal Lowpass

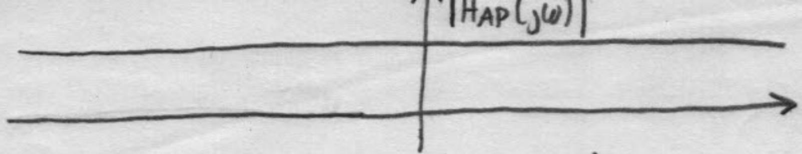


Ideal Highpass

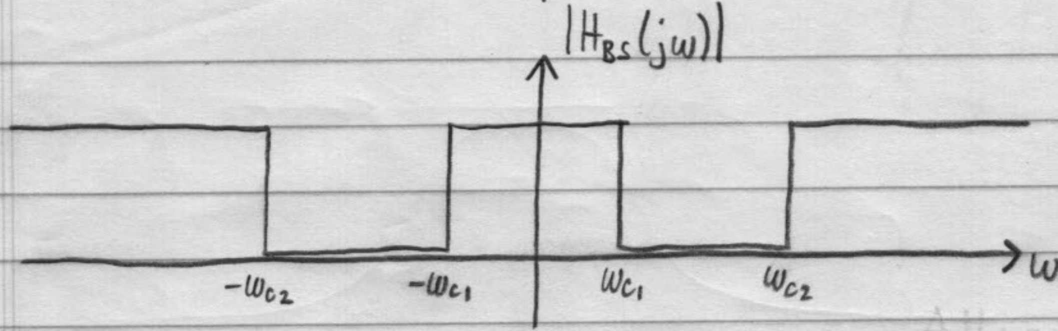


Ideal Bandpass



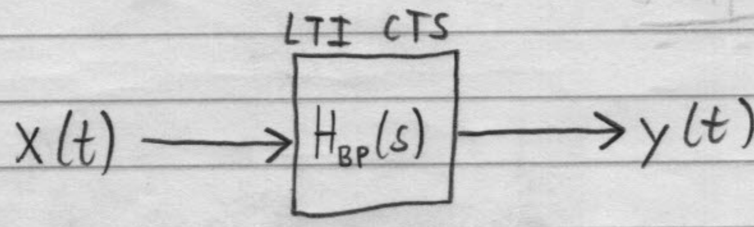


Ideal
Allpass Filter

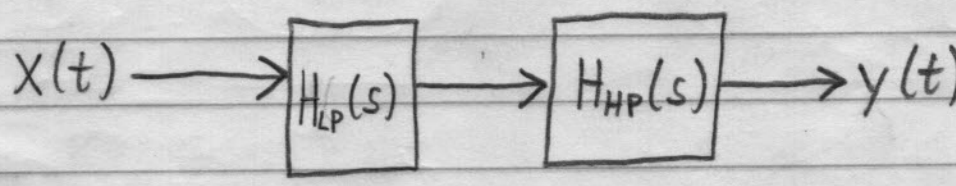


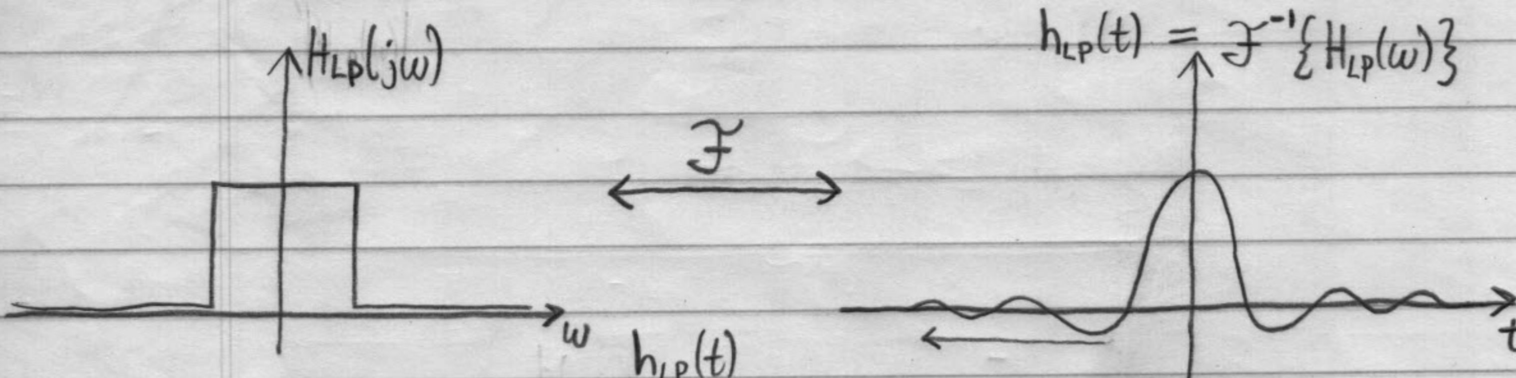
Ideal
Bandstop
(Bandreject)
(Notch)

Cascading
Filters



$$H_{BP}(s) = H_{LP}(s) \cdot H_{HP}(s)$$





Not Realizable

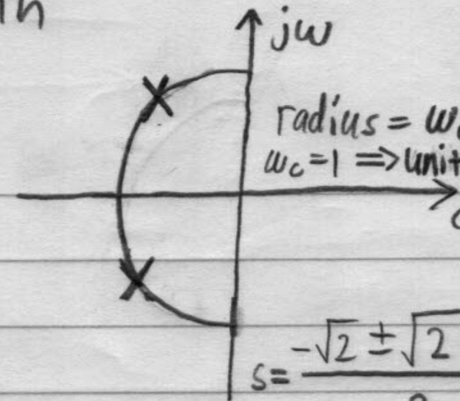
Optimize 3 parameters for a certain filter order $n = \#$ of poles of filter

1.) Passband Flatness $\xrightarrow{\text{maximize}}$ Butterworth

2.) Transition Speed $\xrightarrow{\text{maximize}}$ Chebyshev

3.) Passband Phase Linearity $\xrightarrow{\text{maximize}}$ Bessel
(constant group delay)

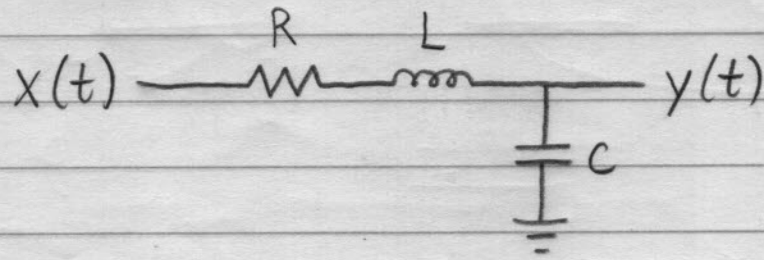
2^{nd} Order Butterworth Filter Example

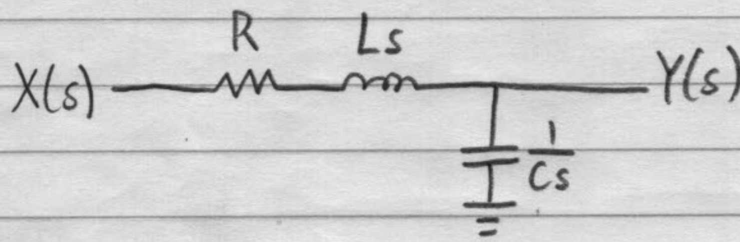


Lowpass Filter Prototype
$\omega_c = 1$ rad/s

$$H(s) = \frac{1}{1 + \sqrt{2}s + s^2}$$

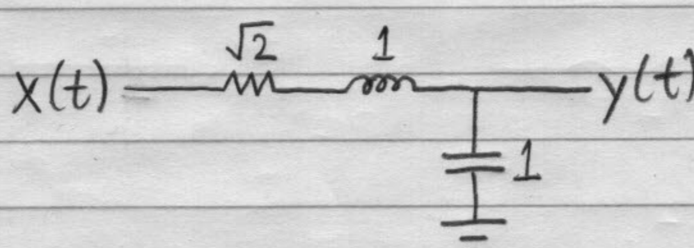
$$s = \frac{-\sqrt{2} \pm \sqrt{2-4}}{2} = \frac{-\sqrt{2} \pm \sqrt{2}j}{2} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$$

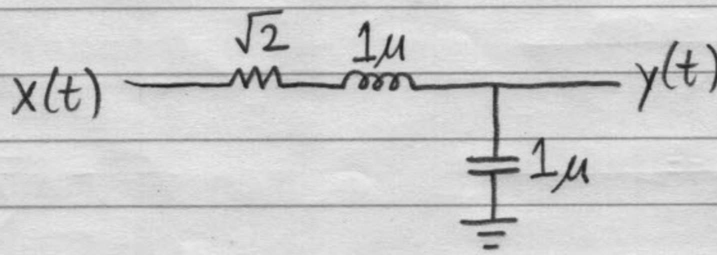




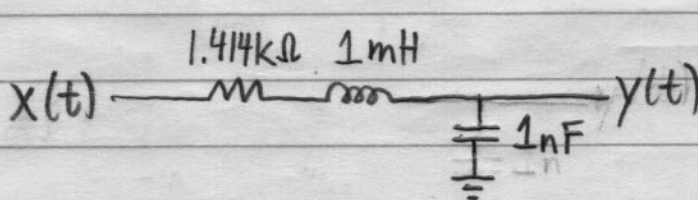
$$H(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R + Ls} = \frac{1}{1 + RCs + CLs^2}$$

$RC = \sqrt{2}$ $\quad$ $CL = 1$ $\Rightarrow$ $C = 1$, $L = 1$, $R = \sqrt{2}$



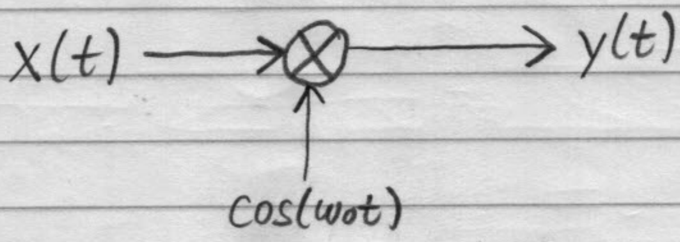


Frequency Scaling to $\omega_c = 1M$ rad/s



Magnitude Scaling

13.) AM Modulation



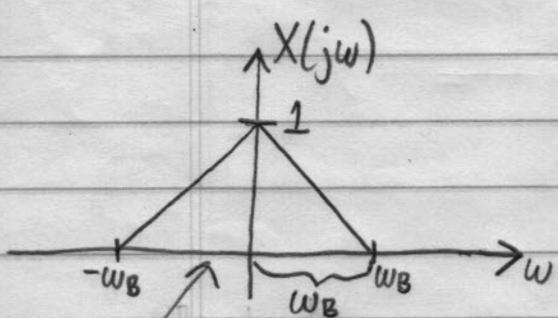
$$y(t) = x(t)\cdot\cos(\omega_0 t)$$

Multiplication Property: $x_1(t)\cdot x_2(t) \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi} X_1(j\omega) * X_2(j\omega)$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * \pi\left[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right]$$

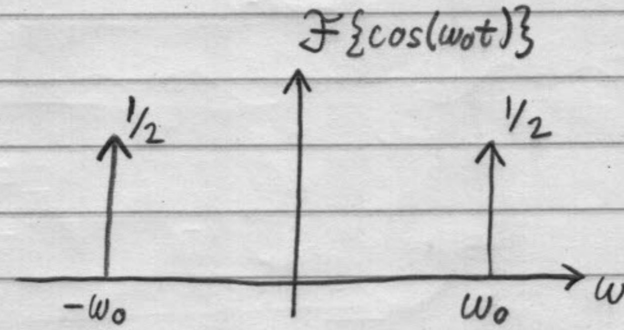
$$= \frac{1}{2}\left[X(j\omega) * \delta(\omega+\omega_0) + X(j\omega) * \delta(\omega-\omega_0)\right]$$

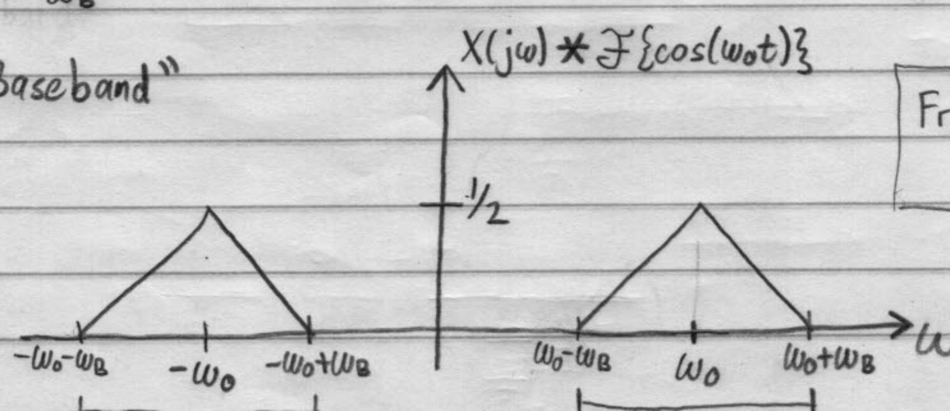
$$= \frac{1}{2} X(j(\omega+\omega_0)) + \frac{1}{2} X(j(\omega-\omega_0))$$



Signal at "Baseband"

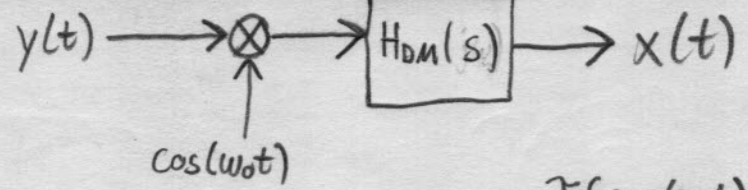
$*$

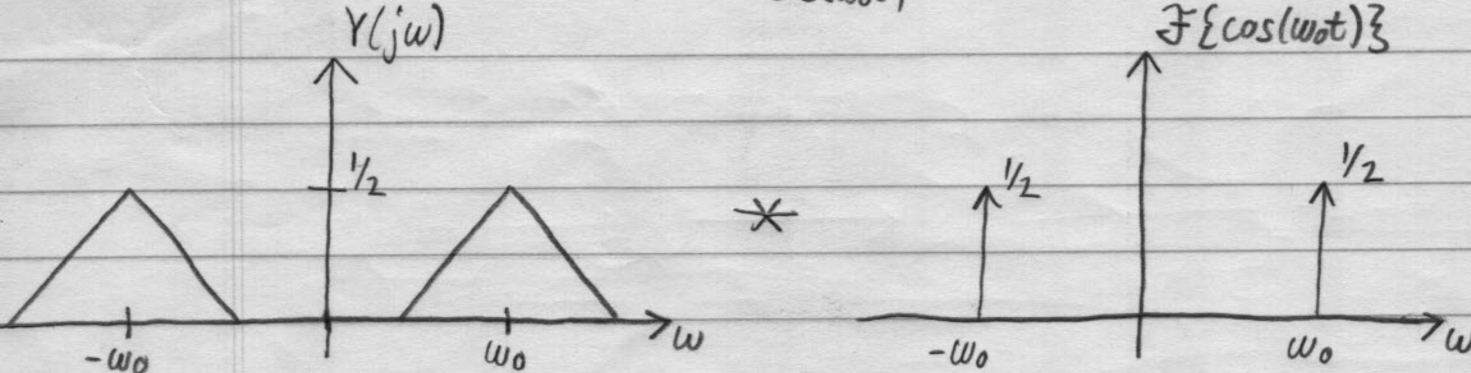


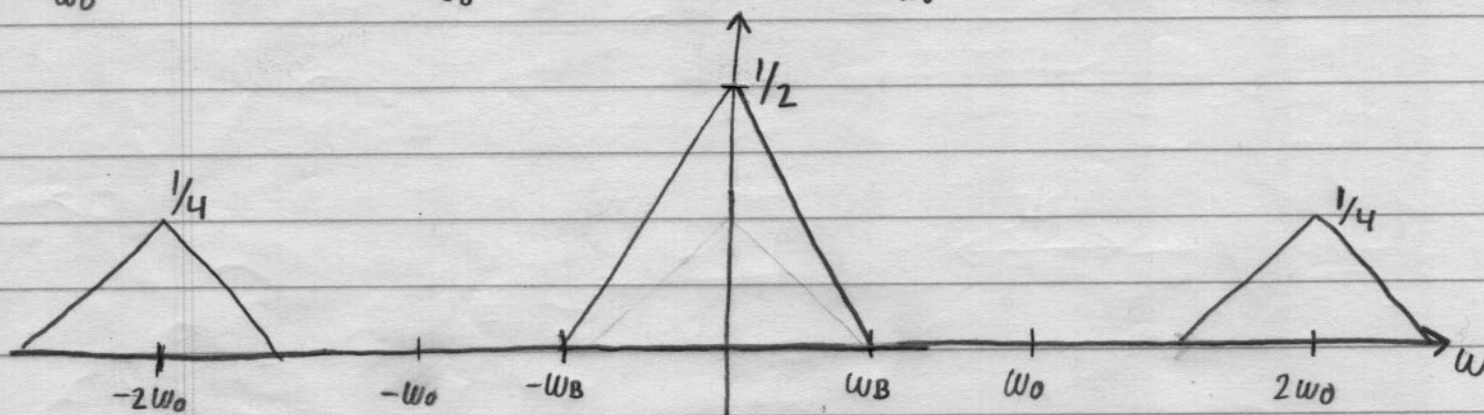


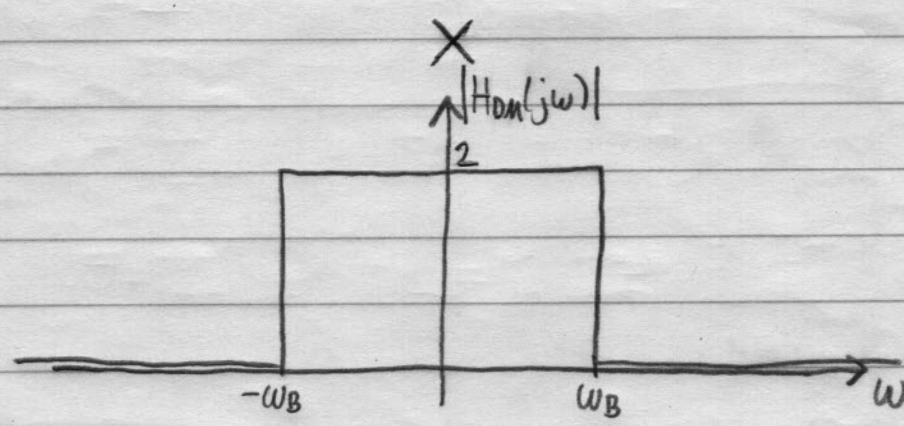
Frequency Division Multiplexing

Demodulation :

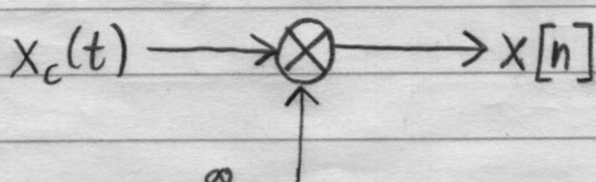




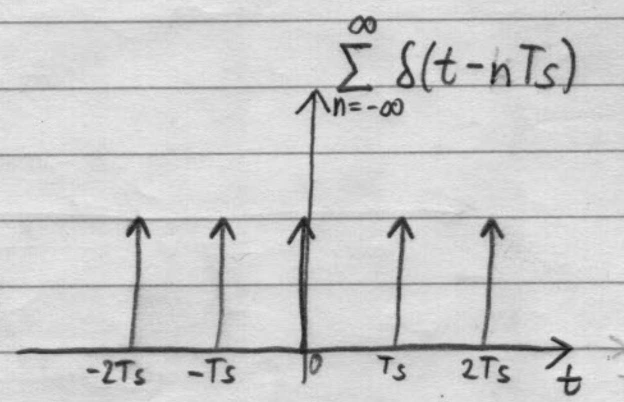




14.) Sampling Theorem



$$\sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$



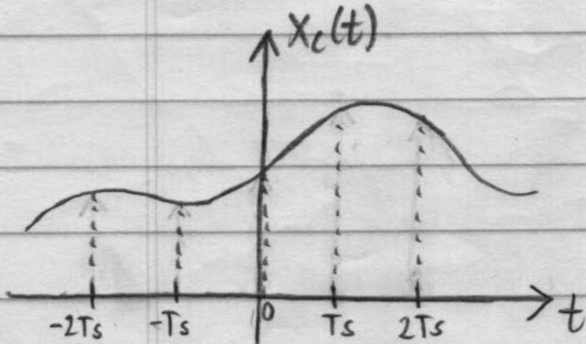
T_s = sampling time

ω_s = sampling frequency

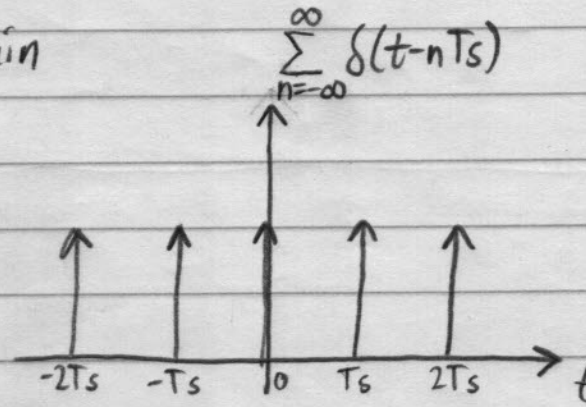
$$x[n] = x_c(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} x_c(t)\,\delta(t-nT_s)$$

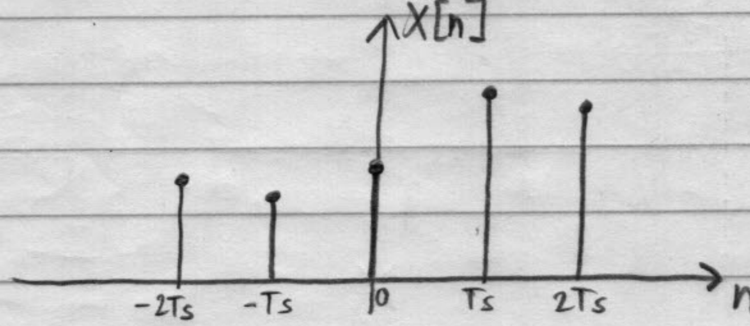
$$= \sum_{n=-\infty}^{\infty} x_c(nT_s)\,\delta(t-nT_s)$$

Time Domain



$\times$

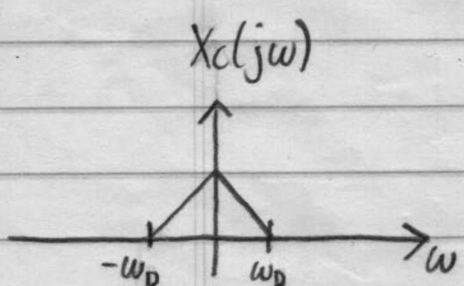




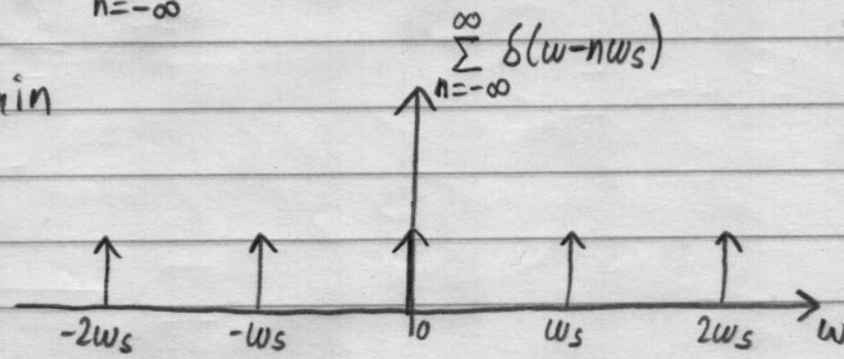
$$\frac{1}{2\pi} X_c(j\omega) * \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

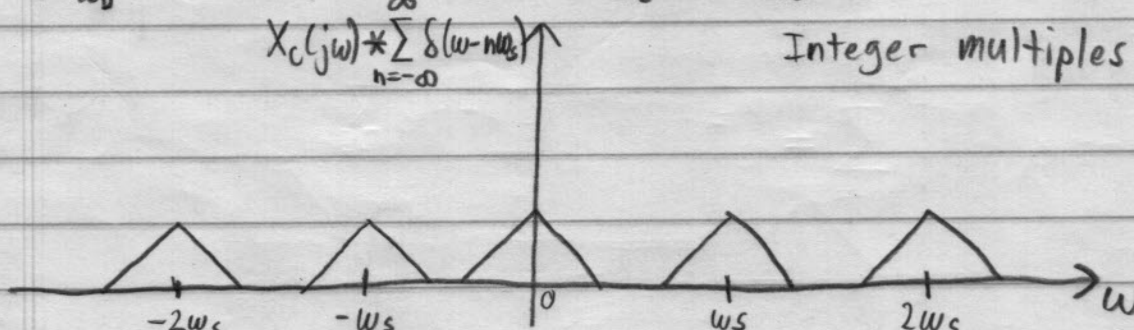
$$= \frac{1}{T_s} X_c(j\omega) * \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

Freq Domain

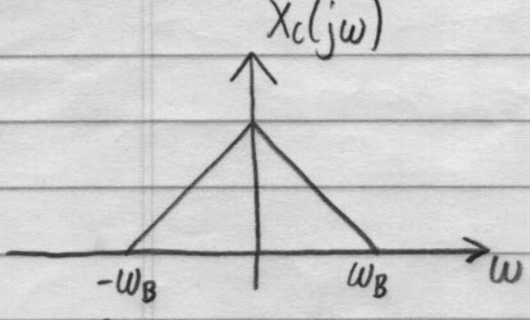


$*$

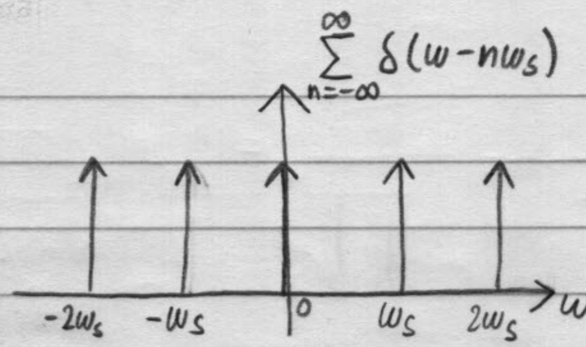




Integer multiples of ω_s

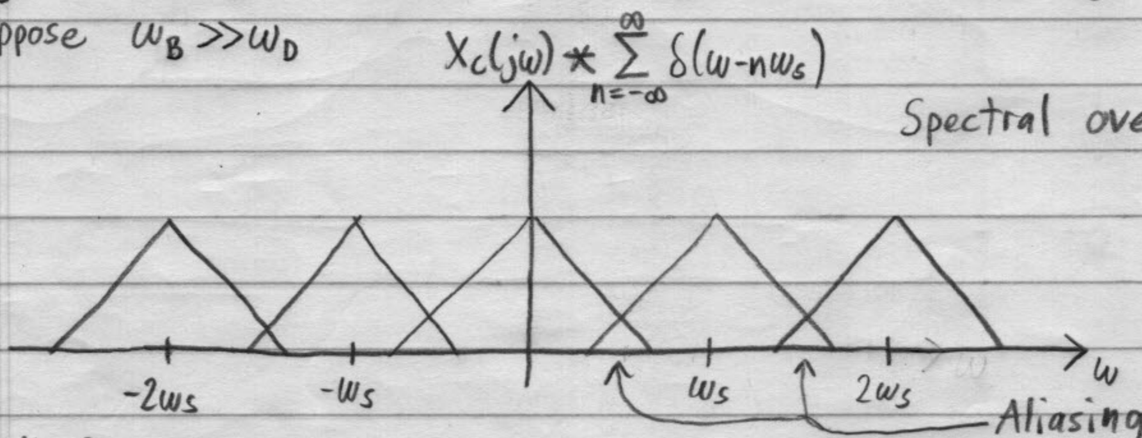


$*$



suppose $\omega_B \gg \omega_D$

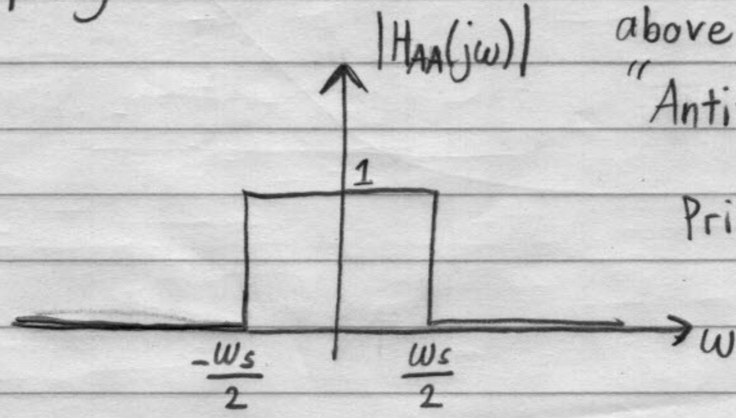
Spectral overlap



To avoid aliasing, we must satisfy: $\rightarrow$ $\boxed{\omega_s > 2\omega_B}$

Nyquist-Shannon Sampling Theorem

$\boxed{\omega_s = 2\omega_B}$ Nyquist Rate



Filter out frequencies above ω_B by using an "Anti-Aliasing Filter" Prior to sampling